Online Appendix

"A Behavioral Theory of Discrimination in Policing"

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A Racial Profiling and the Geography of Policing

The analysis in the main text assumes that police officers decide to allocate their time between policing two groups of people. In the United States, the prevailing law is unclear about whether such group-based profiling is permissible (for an extended discussion, see Knowles, Persico, and Todd 2001). However, under the U.S. Constitution, policies that explicitly treat members of protected categories differently are subject to strict scrutiny (see *Brown v. Board of Education of Topeka*, 1954). A policy of explicitly using group membership to allocate policing resources is not likely to survive a strict scrutiny legal analysis.

We focus on this simple, but potentially illegal, decision-making process in text because it allows us to more clearly focus on our core arguments. However, it can be microfounded with a more complex model where a police chief decides how many policing resources to devote to two neighborhoods: 1 and 2. Formally, assume he devotes n_1 of his time to policing neighborhood 1 and $n_2 = 1 - n_1$ of his time to policing neighborhood 2. Also assume that each neighborhood is comprised of members of the two groups, A and B. Within a neighborhood i, we assume that police interact with a member of group A with probability α_i and a member of group B with probability $1 - \alpha_i$. If police encounters with residents are random and iid, then one way to interpret α_i is that it represents the proportion of neighborhood i that is comprised of members of group A. However, our flexible specification allows for the possibility that police come into contact with members of one group at a rate disproportionate to that group's share of the local population. (Although note that if α_i does not reflect the demographic makeup of neighborhood i, then we simply reintroduce concerns about racial profiling that motivate this microfoundation, just at a different point in the analysis.)

Conditional on a choice about how intensely to police each neighborhood, the share of group A individuals the police encounters is $\eta_A = n_1\alpha_1 + (1-n_1)\alpha_2 = \alpha_2 + (\alpha_1 - \alpha_2)n_1$ and the share of group B individuals the police encounters is $\eta_B = n_1(1-\alpha_1) + (1-n_1)(1-\alpha_2) = 1 - \eta_A$. Recall from the main text that w_A is defined as the share of time that the police officer devotes to policing group A, and $w_B = 1 - w_A$ is the corresponding share of time that the police officer devotes to policing group B. Then, η_A is equivalent to w_A and η_B is equivalent to w_B, and n_1 is a perfect proxy for w_A. More specifically, if police come into contact with group A more than group B in neighborhood 1 (alt. neighborhood 2), $\alpha_1 > \alpha_2$ (alt. $\alpha_1 < \alpha_2$), then increasing n_1 (alt. n_2) linearly increases w_A. Notice that in the extreme cases where $n_1 = 0$ and $n_1 = 1$, then $\eta_A = \alpha_2$ and $\eta_A = \alpha_1$, respectively. Then, α_1 and α_2 correspond the maximum and minimum possible allocations: $\underline{w} = \min\{\alpha_1, \alpha_2\}$ and $\overline{w} = \max\{\alpha_1, \alpha_2\}$.

In a model where police choose n_1 (and not w_A), the analysis in the main text is identical after substituting $\eta_A = \alpha_2 + (\alpha_1 - \alpha_2)n_1$ for w_A.

B Stability in the Multiple Officer Model

The first two equilibrium conditions for the two officer model can be combined as:

$$F_1(w_{A,1}, w_{A,2}) \equiv w_A^{\text{br}}(r_{t,1}, \tilde{r}_{p,1}(w_{A,1}, w_{A,2}, \nu_1)) - w_{A,1} = 0$$
$$F_2(w_{A,1}, w_{A,2}) \equiv w_A^{\text{br}}(r_{t,2}, \tilde{r}_{p,2}(w_{A,1}, w_{A,2}, \nu_2)) - w_{A,2} = 0$$

Close to an equilibrium, we want that for any "small" perturbation to both players' strategies, if the officers iteratively choose best responses given their new beliefs, then the joint allocation would move back to the equilibrium. By standard results in the study of dynamic systems (e.g., Theorem 11.4 in Gintis 2009), this can be expressed by conditions on the matrix of the partial derivatives of the F_i functions:

Definition 3. *Let*

$$D(w_{A,1}, w_{A,2}) = \begin{bmatrix} \frac{\partial F_1}{\partial w_{A,1}} & \frac{\partial F_1}{\partial w_{A,2}} \\ \frac{\partial F_2}{\partial w_{A,1}} & \frac{\partial F_2}{\partial w_{A,2}} \end{bmatrix}.$$

an equilibrium in the two-officer model is is stable if:

(i) *tr*$(D(w^*_{A,1}, w^*_{A,2})) < 0$*, and*

(ii) *det*$(D(w^*_{A,1}, w^*_{A,2})) > 0$*.*

The first condition simplifies to

$$\left.\frac{\partial F_1}{\partial w_{A,1}}\right|_{w_A=w_A^*} + \left.\frac{\partial F_2}{\partial w_{A,1}}\right|_{w_A=w_A^*} < 0$$

Note that if both derivatives are negative (as required in the single officer model), this is always true.

The second condition becomes:

$$\left[\frac{\partial F_1}{\partial w_{A,1}}\frac{\partial F_2}{\partial w_{A,2}} - \frac{\partial F_1}{\partial w_{A,2}}\frac{\partial F_2}{\partial w_{A,1}}\right]_{w_A=w_A^*} > 0$$

To provide a more easily interpretable version of these conditions, define:

$$\begin{aligned} Y_i &= \left.\frac{\partial w_A^{\text{br}}}{\partial \tilde{r}_{p,i}}\right|_{\tilde{r}_{p,i}=\tilde{r}_{p,i}(w_{A,1}^*,w_{A,2}^*)} \\ Z_i &= \left.\frac{\partial \tilde{r}_{p,i}(w_{A,1},w_{A,2})}{\partial w_{A,1}}\right|_{w_A=w_A^*} = \left.\frac{\partial \tilde{r}_{p,i}(w_{A,1},w_{A,2})}{\partial w_{A,2}}\right|_{w_A=w_A^*}. \end{aligned}$$

Then:

$$\left.\frac{\partial F_i}{\partial w_{A,i}}\right|_{w_A=w_A^*} = (Y_iZ_i - 1) \qquad\qquad \left.\frac{\partial F_i}{\partial w_{A,-i}}\right|_{w_A=w_A^*} = Y_iZ_i$$

Plugging these into first stability condition gives:

$$(Y_1Z_1 - 1) + (Y_2Z_2 - 1) < 0 \Longleftrightarrow Y_1Z_1 + Y_2Z_2 < 2 \tag{4}$$

and the second:

$$\begin{aligned} (Y_1Z_1 - 1)(Y_2Z_2 - 1) - (Y_1Z_1)(Y_2Z_2) > 0 \\ \Longleftrightarrow Y_1Z_1 + Y_2Z_2 < 1 \end{aligned}$$

which is stronger than condition (4) and hence the binding constraint.

An intuition for this condition is that due to the complementarities between action and belief, the deviations that are most apt not to return to an equilibrium are those where both officers increase or both officers decrease their allocations. And $Y_1Z_1 + Y_2Z_2$ is the marginal change in the best response as *both* officers increase their allocation to group A. So, this condition states that if both officers were to allocate slightly more time to group A or both allocated slightly less, their best responses would move back toward the equilibrium allocation.

C Proofs

Proof of Lemma 1 Since u is homogeneous with positive degree, for any α there exists a $k > 0$ such that:

$$u(\alpha t_A p_A w_A, \alpha t_B p_B(1-w_A)) = \alpha^k u(t_A p_A w_A, t_B p_B(1-w_A)) \tag{5}$$

Let $\alpha = 1/(t_B p_B)$, and note that w_A maximizes u if and only if it maximizes $(t_B p_B)^{-k} u$. Plugging this into equation (5) gives:

$$(t_B p_B)^{-k} u(t_A p_A w_A, t_B p_B(1-w_A)) = u(r_t r_p w_A, 1-w_A)$$

So, any interior solution w_A^{br} is characterized by the first order condition:

$$\frac{\partial u}{\partial w_A} = r_t r_p u_1(r_t r_p w_A, 1-w_A) - u_2(r_t r_p w_A, 1-w_A) = 0$$

The second derivative is

$$\begin{aligned}\frac{\partial^2 u}{\partial w_A^2} =& r_t r_p(r_t r_p u_{11}(r_t r_p w_A, 1-w_A) - u_{12}(r_t r_p w_A, 1-w_A)) \\ &- r_t r_p u_{12}(r_t r_p w_A, 1-w_A) + u_{22}(r_t r_p w_A, 1-w_A) < 0\end{aligned}$$

the u_{11} and u_{22} terms are strictly negative and the u_{12} are equal to zero by Assumption 1. If we loosen the particular assumption on u_{12}, then the inequality holds as long as the cross-partial derivative is not too negative (relative to the u_{11} and u_{22} terms). Since the objective function is globally strictly concave in w_A, and since it is continuous on a compact set, it must have a unique maximizer.

We now prove part (i) of the lemma. Since u is homogeneous degree k, u_1 is homogeneous degree $k-1$, so we can rewrite the first term of the FOC to give:

$$G(r_t, r_p, w_A) = (r_t r_p)^k u_1(w_A, (r_t r_p)^{-1}(1-w_A)) - u_2(r_t r_p w_A, 1-w_A) = 0 \tag{6}$$

Where w_A^{br} is interior, the change with respect to r_t is given by implicitly differentiating G

$$\frac{\partial w_A^{\text{br}}}{\partial r_t} = -\frac{\frac{\partial G}{\partial r_t}}{\frac{\partial G}{\partial w_A}}$$

The denominator is negative at any maximizer, and the numerator is:

$$\frac{\partial G}{\partial r_t} = kr_p^k r_t^{k-1} u_1(w_A, (r_t r_p)^{-1}(1-w_A)) \\ - (r_t r_p)^k (u_{12}(w_A, (r_t r_p)^{-1}(1-w_A)) r_t^{-2} - r_p w_A u_{12}(r_t r_p w_A, 1-w_A)$$

The first term is strictly positive, and the second two drop out since $u_{12} = 0$. As long as u_{12} is not too positive, then $\frac{\partial w_A^{\text{br}}}{\partial r_t} > 0$. So, at any interior solution, the optimal allocation is strictly increasing in r_t, and since the FOC is strictly increasing in r_t the optimizer is weakly increasing in r_t even when there is a corner solution.

As r_t and r_p enter into the utility symmetrically, the proof for $\frac{\partial w_A^{\text{br}}}{\partial r_p} > 0$ follows an identical logic.

We now prove part (ii) of the lemma. Using the symmetry property from Assumption 1, $u(x, y) = u(y, x)$ implies $u_1(x, y) = u_2(y, x)$. The FOC when $r_t r_p = 1$ is

$$u_1(w_A, 1-w_A) = u_2(w_A, 1-w_A)$$

which is clearly met at $w_A = 1/2$.

Proof of Lemma 2 The proof of Lemma 1 shows that the first derivative of the objective function is continuous and strictly decreasing in w_A. So, there will be an interior solution if and only if it is strictly positive at $w_A = \underline{w}$ and strictly negative at $w_A = \overline{w}$. The first condition requires:

$$r_t r_p u_1(r_t r_p \underline{w}, 1-\underline{w}) > u_2(r_t r_p \underline{w}, 1-\underline{w}) \\ r_t r_p > \frac{u_2(r_t r_p \underline{w}, 1-\underline{w})}{u_1(r_t r_p \underline{w}, 1-\underline{w})}$$

Similarly, the second condition requires:

$$r_t r_p < \frac{u_2(r_t r_p \overline{w}, 1-\overline{w})}{u_1(r_t r_p \overline{w}, 1-\overline{w})}$$

Combining gives the result.

Proof of Proposition 3. If $w_A^{\text{br}}(r_t, \tilde{r}_p(\underline{w}+\epsilon)) = \underline{w}$ for some $\epsilon > 0$ or $w_A^{\text{br}}(r_t, \tilde{r}_p(\overline{w}-\epsilon)) = \overline{w}$ for some $\epsilon > 0$, then there is a stable corner equilibrium allocation. To complete the proof we need to show that if neither of these hold, there is an interior equilibrium. Let

$$F(w_A) = w_A^{\text{br}}(r_t, \tilde{r}_p(w_A)) - w_A \tag{7}$$

That is, $F(w_A)$ represents how he would change his allocation if starting from w_A, and an equilibrium is a point where $F(w_A^*) = 0$. If there is no stable corner solution, then it must be the case that $w_A^{\text{br}}(r_t, \tilde{r}_p(\underline{w} + \underline{\epsilon})) > \underline{w}$ for some small $\underline{\epsilon} \in (0, 1/2)$, and hence $F(\underline{w} + \underline{\epsilon}) > 0$. There must also be a $\overline{\epsilon} \in (0, 1/2)$ such that and $w_A^{\text{br}}(r_t, \tilde{r}_p(\overline{w} - \overline{\epsilon})) > 0$ and similarly $F(\overline{w} - \overline{\epsilon}) < 0$. By the continuity of w_A^{br} in $\tilde{r}_p$ and the continuity of $\tilde{r}_p$ in w_A, F is continuous in w_A, and so the intermediate value theorem implies there must be a $w_A^* \in (\underline{\epsilon}, \overline{\epsilon})$ such that $F(w_A^*) = 0$, where $F'(w_A) < 0$. Finally, since $F'(w_A) = \frac{\partial w_A^{\text{br}}}{\partial w_A} - 1$, then $F'(w_A^*) < 0 \iff \left.\frac{\partial w_A^{\text{br}}}{\partial w_A}\right|_{w_A = w_A^*} < 1$, and w_A^* is stable. ∎

In the main text, we describe the following result. Here, we state and prove it formally.

Proposition 6. *If the officer utility is given by equation 1, then there is a unique equilibrium in which the officer chooses a policing allocation*

$$w_A^* = \begin{cases} \underline{w} & \text{if } \widehat{w}_A < \underline{w} \\ \widehat{w}_A & \text{if } \widehat{w}_A \in [\underline{w}, \overline{w}] \\ \overline{w} & \text{if } \widehat{w}_A > \overline{w} \end{cases}$$

where

$$\widehat{w}_A = w_A^\dagger + \frac{\nu(r_t r_p - 1)}{(1-\nu)(1 + r_t r_p)} \tag{8}$$

and forms a (potentially inaccurate) belief $\tilde{r}_p^*$ using (2).

Proof of Proposition 6 Using Definition 1,an equilibrium policing allocation w_A solves

$$w_A^* = w_A^{\text{br}}(r_t, \tilde{r}_p^*(w_A^*))$$

At any interior solution, $w_A^{\text{br}}(r_t, r_p) = \frac{r_t \tilde{r}_p(w_A)}{1 + r_t \tilde{r}_p(w_A)}$. Substituting (2) and solving this equation for w_A gives a unique solution $\widehat{w}_A$, defined by equation (8) in the main text. Thus when $\widehat{w}_A$ lies in $[\underline{w}, \overline{w}]$ it meets the condition for a unique equilibrium allocation, $w_A^* = \widehat{w}_A$.

To prove that the corner solutions lie where the proposition claims, it helps to first describe the shape of the function which in turn describes how the allocation would change if playing an unconstrained best response starting at w_A,

$$F(w_A) = \frac{r_t \tilde{r}_p(w_A)}{1 + r_t \tilde{r}_p(w_A)} - w_A,$$

on the full range of $[0, 1]$. This function is continuous and differentiable. It is immediate that

$F(0) = 0$ and $F(1) = 0$,[6] and by the analysis above $F(\widehat{w}_A) = 0$. So, when $\widehat{w}_A \in (0, 1)$, there are three zeroes on $[0, 1]$, and when $\widehat{w}_A$ lies outside of this interval the only zeroes are at the endpoints (and hence the function must be always positive or negative). Recall that:

$$\widehat{w}_A = \frac{r_t r_p}{1 + r_t r_p} + \frac{\nu(r_t r_p - 1)}{(1 - \nu)(1 + r_t r_p)}$$

Rearranging and simplifying gives:

$$0 < \widehat{w}_A < 1 \iff \nu < r_t r_p < 1/\nu$$

In order to see whether F is positive or negative as $w_A \to 0$ and $w_A \to 1$, we need to check F' at these two points. Taking the first derivative of F yields:

$$F'(w_A) = \frac{\nu r_p r_t \left(2(1-\nu) w_A^2 - 2(1-\nu) w_A + 1\right)}{\left(\nu w_A^2 \left(r_p r_t + 1\right) - 2\nu w_A + \nu + (1 - w_A) w_A \left(r_p r_t + 1\right)\right)^2} - 1$$

Evaluating at 0 and 1 gives:

$$F'(0) > 0 \iff r_t r_p > \nu \qquad\qquad F'(1) > 0 \iff r_p r_t < \frac{1}{\nu}$$

Since $\nu < 1/\nu$, there are three cases we must consider, corresponding to three possible shapes of the F function. In case (I), $r_t r_p \geq 1/\nu$. When the inequality is strict, this implies F is increasing at 0, decreasing at 1, and has no interior root, and hence $F(w_A) > 0$ for $w_A \in (0, 1)$. When $r_t r_p = 1/\nu$, the only difference is that $F'(1) = 0$, but F is decreasing for w_A approaching 1, and this does not affect the rest of the argument. In case (II), $\nu < r_t r_p < 1/\nu$, and so F is increasing at 0 and at 1, with an interior zero at $\widehat{w}_A$, and hence $F(w_A) > 0$ for $w_A \in (0, \widehat{w}_A)$ and $F(w_A) < 1$ for $w_A \in (\widehat{w}_A, 1)$. In case (III) $r_t r_p \leq \nu$, and F is decreasing at 0 (or, in the case where $r_t r_p = \nu$, flat at 0 but decreasing for small w_A), increasing at 1, and has no interior root, and hence $F(w_A) < 0$ for $w_A \in (0, 1)$. Note that there can only be an interior equilibrium in case (II), and it must be the case that $F'(\widehat{w}_A) < 0$, which is equivalent to, the stability condition.

Now we can complete proving where the equilibrium lies and uniqueness when the domain of the allocation choice is restricted to $[\underline{w}, \overline{w}]$. If $\widehat{w}_A \leq 0$ then the F function is in case (III) above, and so $F(\underline{w}) < 0$. If $0 < \widehat{w}_A < \underline{w}$, it is in case (II), but since $\underline{w} \in (\widehat{w}_A, 1)$ it must also be the case that $F(w_A) < 0$ for all $w_A \in [\underline{w}, \overline{w}]$. And returning to the definition of w_A^{br}, $F(\underline{w}) < 0$, implies $w_A^{\text{br}}(r_t, \tilde{r}_p(\underline{w})) = \underline{w}$, meaning there is an extreme equilibrium at $\underline{w}$. $F(w_A) < 0$ also implies there is no interior equilibrium or equilibrium at $\overline{w}$ since $F(\overline{w}) < 0$, so this equilibrium is unique. If

[6] This implies that if we did not restrict the range to $[\underline{w}, \overline{w}]$, there would always be an equilibrium only policing either group, though this would not meet the stability condition whenever an interior equilibrium exists.

$\widehat{w}_A = \underline{w}$, then it is immediate that $F(\underline{w}) = \underline{w}$, and hence there is an extreme equilibrium at this bound, and this equilibrium is unique since $F(w_A) < 0$ for $w_A \in (\underline{w}, \overline{w}]$.

When $\underline{w} < \widehat{w}_A < \overline{w}$, $\widehat{w}_A$ is an interior equilibrium, and there can't be another interior state since there is no other point on $[\underline{w}, \overline{w}]$ where $F(w_A) = 0$. The F function is in case (II), which implies $F(\underline{w}) > 0$ and $F(\overline{w}) < 0$, so there is no equilibrium at the extremes. Thus the equilibrium is unique.

By a similar argument to the $\widehat{w}_A \leq \underline{w}$ case, if $\widehat{w}_A \geq \overline{w}$, then $w_A^{\text{br}}(r_t, \tilde{r}_p(\overline{w})) = \overline{w}$, and there can't be an equilibrium at $\underline{w}$ or on the interior. ∎

Proof of Proposition 4. Let $\nu \in (0, 1)$.

Part (i) immediately follows from the facts that $w_A^{\text{br}}(1, 1) = 1$ and $\tilde{r}_p(1/2) = 1$.

For part (ii) as in the proof of Proposition 3 let $F(w_A)$ be the difference between w_A and the best response allocation give the belief generated by w_A. If the equilibrium is unique, it must be stable by Proposition 3. And so if w_A^* is the equilibrium, it must be the case that $F(w_A) > 0$ if and only if $w_A < w_A^*$ and $F(w_A) < 0$ if and only if $w_A > w_A^*$.

From Lemma 1, if $r_t r_p < 1$ then $w_A^\dagger < 1/2$, and so $\tilde{r}_p(w_A^\dagger) < r_p$, and so $w_A^{\text{br}}(r_t, r_p) > w_A^{\text{br}}(r_t, \tilde{r}_p(w_A^\dagger))$, and $F(w_A^\dagger) < 0$. Therefore $w_A^\dagger > w_A^*$, and $\tilde{r}_p(w_A^*) < r_p$. The proof for $r_t r_p > 1$ follows an identical logic. ∎

Proof of Proposition 5. To prove the existence of an equilibrium allocation, define a function $G : [\underline{w}, \overline{w}]^2 \to [\underline{w}, \overline{w}]^2$ given by

$$G(w_{A,1}, w_{A,2}) \equiv (w_A^{\text{br}}(r_{t,1}, \tilde{r}_{p,1}(w_{A,1}, w_{A,1}), w_A^{\text{br}}(r_{t,2}, \tilde{r}_{p,2}(w_{A,1}, w_{A,1})).$$

This is a continuous mapping from a compact and convex set to itself, so by the Brouwer fixed point theorem there must be a $(w_{A,1}^*, w_{A,2}^*)$, such that $G(w_{A,1}^*, w_{A,2}^*) = (w_{A,1}^*, w_{A,2}^*)$, which is an equilibrium allocation, with corresponding equilibrium beliefs given by $\tilde{r}_{p,i}^* = \tilde{r}_{p,i}(w_{A,1}^*, w_{A,2}^*)$.

We now show the comparative static results. First, recall we can write the equilibrium conditions as the following system of equations:

$$F_1(w_{A,1}, w_{A,2}; r_{t,1}, \nu_1) = w_A^{\text{br}}(r_{t,1}, \tilde{r}_{p,1}(w_{A,1}, w_{A,2})) - w_{A,1} = 0$$
$$F_2(w_{A,1}, w_{A,2}; r_{t,1}, \nu_1) = w_A^{\text{br}}(r_{t,2}, \tilde{r}_{p,2}(w_{A,1}, w_{A,2})) - w_{A,2} = 0$$

For part (i), we prove the result as $r_{t,1}$ changes, but identical logic holds for $r_{t,2}$.

To implicitly differentiate the equilibrium conditions with respect to $r_{t,1}$, take the total derivative of F_1 and F_2 (at w_A^*, accounting for the fact that $w_{A,i}$ are a function of $r_{t,1}$:

$$\left.\frac{dF_1}{dr_{t,1}}\right|_{w_A=w_A^*} = \left(\left.\frac{\partial w_A^{\text{br}}}{\partial r_{t,1}}\right|_{w_A=w_A^*} + Y_1 \left(Z_1 \left.\frac{\partial w_{A,1}}{\partial r_{t,1}}\right|_{w_A=w_A^*} + Z_1 \left.\frac{\partial w_{A,2}}{\partial r_{t,1}}\right|_{w_A=w_A^*} \right) \right) - \left.\frac{\partial w_{A,1}}{\partial r_{t,1}}\right|_{w_A=w_A^*} = 0 \tag{9}$$

$$\left.\frac{dF_2}{dr_{t,1}}\right|_{w_A=w_A^*} = \left(Y_1 \left(Z_2 \left.\frac{\partial w_{A,1}}{\partial r_{t,1}}\right|_{w_A=w_A^*} + Z_2 \left.\frac{\partial w_{A,2}}{\partial r_{t,1}}\right|_{w_A=w_A^*} \right) \right) - \left.\frac{\partial w_{A,2}}{\partial r_{t,1}}\right|_{w_A=w_A^*} = 0 \tag{10}$$

where as in section B we define:

$$\begin{aligned} Y_i &= \left.\frac{\partial w_A^{\text{br}}}{\partial \tilde{r}_{p,i}}\right|_{\tilde{r}_{p,i}=\tilde{r}_{p,i}(w_{A,1}^*, w_{A,2}^*)} \\ Z_i &= \left.\frac{\partial \tilde{r}_{p,i}(w_{A,1}, w_{A,2})}{\partial w_{A,1}}\right|_{w_A=w_A^*} = \left.\frac{\partial \tilde{r}_{p,i}(w_{A,1}, w_{A,2})}{\partial w_{A,2}}\right|_{w_A=w_A^*}. \end{aligned}$$

Equations (9) and (10) are a system of two equations where we want to solve for $\frac{\partial w_{A,1}}{\partial r_{t,1}}$ and $\frac{\partial w_{A,1}}{\partial r_{t,1}}$. Define the following:

$$T_1 = \left.\frac{\partial w_{A,1}}{\partial r_{t,1}}\right|_{w_A=w_A^*} \qquad T_2 = \left.\frac{\partial w_{A,2}}{\partial r_{t,1}}\right|_{w_A=w_A^*} \qquad X = \left.\frac{\partial w_A^{\text{br}}}{\partial r_{t,1}}\right|_{w_A=w_A^*}$$

Then, we can rewrite this system of equations as

$$\begin{aligned} (X + Y_1 Z_1 (T_1 + T_2)) - T_1 &= 0 \\ Y_1 Z_1 (T_1 + T_2) - T_1 &= 0 \end{aligned}$$

and goal is to solve for T_1 and T_2. This gives:

$$\begin{aligned} T_1 &= X + \frac{X Y_1 Z_1}{1 - Y_1 Z_1 - Y_2 Z_2} \\ T_2 &= \frac{X Y_2 Z_2}{1 - Y_1 Z_1 - Y_2 Z_2}. \end{aligned}$$

Since we know that $X > 0$, $Y_i > 0$, and $Z_i > 0$, both of these are strictly positive if and only if $1 - Y_1 Z_1 - Y_2 Z_2 > 0$, which is exactly the stability condition for an interior equilibrium derived in section B. Finally, since $\Delta^* = |w_{A,i}^* - w_{A,i}^\dagger|$ and $w_{A,i}^\dagger$ is constant in $r_{t,1}$, then for each $i \in \{1, 2\}$, Δ^* increases in $r_{t,1}$.

For part (ii), we prove the result as ν_1 changes, but identical logic holds for ν_2. We now define the following:

$$N_1 = \left.\frac{\partial w_{A,1}}{\partial \nu_1}\right|_{w_A = w_A^*} \qquad\qquad N_2 = \left.\frac{\partial w_{A,2}}{\partial \nu_1}\right|_{w_A = w_A^*}$$

To implicitly differentiate the equilibrium conditions with respect to ν_1, take the total derivative of the equilibrium conditions at w_A^*, accounting for the fact that $w_{A,i}$ is a function of ν_1:

$$Y_1\left(Z_1 N_1 + Z_1 N_2 + \frac{\partial \tilde{r}_{p,1}}{\partial \nu_1}\right) - N_1 = 0 \qquad\qquad Y_2\left(Z_2 N_1 + Z_2 N_2\right) - N_1 = 0$$

Our goal is to solve for N_1 and N_2, which gives:

$$N_1 = \frac{\partial \tilde{r}_{p,1}}{\partial \nu_1}\left(\frac{Y_1(1 - Y_2 Z_2)}{1 - Y_1 Z_1 - Y_2 Z_2}\right) \qquad\qquad N_2 = \frac{\partial \tilde{r}_{p,1}}{\partial \nu_1}\left(\frac{Y_1 Y_2 Z_2}{1 - Y_1 Z_1 - Y_2 Z_2}\right)$$

Again since we know that $X > 0$, $Y_i > 0$, and $Z_i > 0$, both of these are strictly positive at an interior equilibrium if and only if the stability condition is met and $\frac{\partial \tilde{r}_{p,1}}{\partial \nu_1} > 0$. This latter condition holds if $w_A = w_{A,1} + w_{A,2} > 1$ (i.e., group A receives a higher allocation than group B). Similarly, if $w_A < 1$, then $\frac{\partial \tilde{r}_{p,1}}{\partial \nu_1} < 0$ and hence both officers police group B more as ν_1 increases. ∎

D Misspecified Beliefs

Here we consider a more general notion of the idea that officers underestimate the effect of their policing decisison on the crime data.

Formally, suppose at the stage where the officer is forming beliefs about the relative crime rates, he does so as if he thinks the crime detection function is $\tilde{c}(w_J, p_J)$, which may not match the real function $c(w_J, p_J)$.[7] In our main results below where the officer has a misspecified model, we will maintain the assumption from above that crimes are detected according to the function $c(w_J, p_J) = w_J p_J$. However, before getting to this, it is instructive to consider how this kind of misspecified model affects belief formation for a general $c(w_J, p_J)$ with the weaker assumption that it is continuously differentiable, weakly increasing in w_J, and strictly increasing in p_J. Further, assume that the officer's belief $\tilde{c}$, while potentially not equal to c, still shares these properties.

[7] There is a tension in literally interpreting this source of misspecification since the officer behaves as if he does know the correct crime function when solving the optimization problem for his allocation, but not when forming beliefs. The simplest resolution, which will be more natural in the model with multiple officers, is that the officer thinks about crime detection differently when choosing his own allocation versus when he forms beliefs based on crime data, which in reality also depends on the choices made by others. Alternatively, what matters for the allocation stage is not the specific functional form but the fact that the optimal allocation is increasing in r_t and r_p. Similarly, what matters in the belief formation stage is not that incorrect beliefs are driven by a misspecified c function, but that beliefs become a function of the allocation.

This assumption implies that, upon observing a crime level c_J, there is a unique value of p_J that solves $c_J = \tilde{c}(w_J, p_J)$.[8] Let $\hat{p}(c_J, w_J)$ be this value of p_J, which is the officer's inference about p_J given the observed crime and allocation data. Given a real crime production function $c_J = c(w_J, p_J)$, we can then write the officer's inference about the crime rate of group J as a function of the real value and the allocation choice, which we write $\tilde{p}_J = \hat{p}(c(w_J, p_J), w_J)$.

We are primarily interested in when there is an interaction between the officer choice w_J and this resulting belief. Fortunately, there is a clean characterization of when such interactions occur.

Proposition 7. *The officer's belief about the crime rate of group J is strictly increasing in w_J if he strictly underestimates the impact of w_J on c_J, and is strictly decreasing in w_J if he strictly overestimates this quantity:*

$$\textit{sign}\left(\frac{\partial \tilde{p}_J}{\partial w_J}\right) = \textit{sign}\left(\frac{\partial c}{\partial w_J} - \frac{\partial \tilde{c}}{\partial w_J}\right)$$

Proof of Proposition 7 Recall that $\hat{p}_J$ is a solution to:

$$G(p_J; c_J, w_J) = \tilde{c}(w_J, p_J) - c_J = 0 \tag{11}$$

and

$$\tilde{p}_J = \hat{p}_J(c(w_J, p_J), w_J).$$

So:

$$\begin{aligned}
\frac{\partial \tilde{p}_J}{\partial w_J} &= \frac{\partial \hat{p}}{\partial w_J} + \frac{\partial \hat{p}}{\partial c}\frac{\partial c}{\partial w_J} \\
&= -\frac{\frac{\partial G}{\partial w_J}}{\frac{\partial G}{\partial p_J}} + -\frac{\frac{\partial G}{\partial c_J}}{\frac{\partial G}{\partial p_J}}\frac{\partial c}{\partial w_J} \\
&= -\frac{\frac{\partial \tilde{c}}{\partial w_J}}{\frac{\partial \tilde{c}}{\partial p_J}} + \frac{1}{\frac{\partial \tilde{c}}{\partial p_J}}\frac{\partial c}{\partial w_J} \\
&= \frac{\frac{\partial c}{\partial w_J} - \frac{\partial \tilde{c}}{\partial w_J}}{\frac{\partial \tilde{c}}{\partial p_J}}
\end{aligned}$$

The numerator is strictly positive, and so the sign of the derivative is equal to the sign of the numerator. ∎

[8] An important implicit assumption here is that the officer observes "enough data" that the crime detected is exactly $c(w_J, p_J)$. That is, we do not explicitly model the randomness inherent to the process. We do so to keep the model simple and the focus on the application; as elaborated when we introduce our full solution concept below, several theoretical papers study the convergence of beliefs and actions when explicitly modeling such randomness.

E More General Beliefs (Multiple Officer Model)

There are several ways one could extend the definition of non-conditioning bias to the multiple officer model. One potentially realistic change would be to assume that officers may do a better (or worse) job of adjusting for their own behavior than others' behavior when forming inferences about the p_J parameters. Formally, we could define the officer belief as:

$$\tilde{r}_{p,i}(w_A) = \frac{\frac{c_A}{\nu_i^s + (1-\nu_i^s) w_{A,i} + \nu_i^o + (1-\nu_i^o) w_{j,2}}}{\frac{c_B}{\nu_i^s + (1-\nu_i^s) w_{B,i} + \nu_i^o + (1-\nu_i^o) w_{B,j}}} \tag{12}$$

where the $\nu_i^s \in [0,1]$ represents how well the officer conditions for his own allocation and $\nu_i^o \in [0,1]$ represents how well he conditions on the other officer choice. A key feature of this more general belief is that as long as $\nu_i^s > 0$, it is increasing in $w_{A,i}$, meaning the officer's belief about A's relative crime rate increases in how much he polices this group. Similarly, as long as $\nu_i^o > 0$, the officer's belief about the relative crime rate of group A increases in how much the other officer polices this group. So, while the the analysis is more complicated with this belief formation, the general feedback loop and spillover dynamics are present here as well.

F Endogenous Crime Rates

Many prior studies have focused on how police behavior affects individuals' propensities to engage in criminal activity (i.e., "deterrence") and/or the strategic interactions between police and potential offenders more generally (e.g., Knowles, Persico, and Todd 2001; Anwar and Fang 2006). In this section we show that the results of the model hold in an extension where crime rates are endogenous to the policing allocations.

We pick a particular functional form which makes the rest of the analysis go through more or less unchanged, though the general logic should extend to more general specifications. Let:

$$p_J(w_J) = p_J^0 w_J^\beta$$

for some $p_J^0 > 0$ and $\beta \in (-1, 0]$. In words, p_J^0 represents the "baseline" crime rate.

The $\beta = 0$ case captures the main analysis, and if $\beta < 0$ the crime rate decreases in w_J.

The $\beta > -1$ constraint is to prevent the case where $p_J(w_J)$ decreases so quickly in $p_J(w_J)$ that $c_J = p_J(w_J) w_J$ is decreasing in w_J. If less crime is caught the more a group is policed more, and the officer goal is to catch crime, then this leads to an unusual dynamic where the officer can want to police group with higher crime rates *less*. However, as discussed below, if the officer's goal is to prevent crime from happening in the first place, the $\beta \leq 1$ case still leads to the key property that officers want to spend more time policing the group with a higher baseline crime rate.

Given this assumption, we can now write the officer utility as:

$$u(c_A, c_B) = u(t_A p_A^0 w_A^{1+\beta}, t_B p_B^0 (1 - w_A)^{1+\beta})$$

Using a similar trick as the proof of Lemma 1, we can multiply each argument by $1/(t_B p_B^0)$ and use the assumption that u is homogeneous of degree k to get that

$$(t_B p_B^0)^{-k} u(t_A p_A^0 w_A^{1+\beta}, t_B p_B^0 (1 - w_A)^{1+\beta}) = u(r_t r_p^0 w_A^{1+\beta}, (1 - w_A)^{1+\beta}) \tag{13}$$

where r_t is defined as in the main model, and $r_p^0 = p_A^0 / p_B^0$ is now the relative *baseline* crime rate.

Since $\beta > -1$, this is increasing and concave in w_A. So there is either a corner solution, or an interior solution characterized by:

$$\frac{\partial u}{\partial w_A} = r_t r_p^0 u_1(r_t r_p^0 w_A^{\beta+1}, (1 - w_A)^{\beta+1})(\beta + 1) w_A^{\beta} - u_2(r_t r_p^0 w_A, (1 - w_A)^{\beta+1})(\beta + 1)(1 - w_A)^{\beta} = 0$$

Let:

$$G(w_A; r_t, r_p^0) = \frac{\partial u}{\partial w_A}.$$

The comparative statics on the optimal allocation are determined by implicitly differentiating this G. For example, the optimal allocation is increasing in r_p^0 if and only if:

$$-\left.\frac{\frac{\partial G}{\partial r_t}}{\frac{\partial G}{\partial w_A}}\right|_{w_A = w_A^*}$$

As long as $\beta > -1$ (as assumed), and $\frac{\partial G}{\partial r_p^0}$ is positive by the same analysis as lemma 1. Similarly, $\frac{\partial G}{\partial r_t} > 0$.

A specific functional form A functional form that generalizes the main example is if:

$$u(c_A, c_B) = (t_A c_A)^{\alpha} + (t_B c_B)^{\alpha}$$

for some $0 < \alpha < 1$, where $\alpha = 1/2$ is the main example. Plugging in the value of c_J with endogenous crime gives:

$$u(w_A) = (t_A p_A^0 w_A^{\beta+1})^{\alpha} + (t_B p_B^0 (1 - w_A)^{\beta+1})^{\alpha} \tag{14}$$

$$= (t_A p_A^0)^{\alpha} w_A^{\alpha(\beta+1)} + (t_B p_B^0)^{\alpha} (1 - w_A)^{\alpha(\beta+1)} \tag{15}$$

Since $0 < \alpha < 1$ and $0 < \beta + 1 < 1$, it follows that $0 < \alpha(\beta + 1) < 1$ and that this expression is concave in w_A. When there is an interior solution it is unique and characterized by $u'(w_A) = 0$, or:

$$
\begin{aligned}
(t_A p_A^0)^\alpha w_A^{\alpha(\beta+1)-1} &= (t_B p_B^0)^\alpha (1 - w_A)^{\alpha(\beta+1)-1} \\
\left(\frac{w_A}{1 - w_A}\right)^{\alpha(\beta+1)-1} &= (r_t r_p^0)^{-\alpha}
\end{aligned}
$$

and so:

$$
w_A^* = \frac{(r_t r_p^0)^\gamma}{1 + (r_t r_p^0)^\gamma} \tag{16}
$$

where $\gamma = \frac{\alpha}{1-\alpha(\beta+1)}$. Note the main example of the baseline model is the case here $\beta = 0$ and $\alpha = 1/2$, in which case $\gamma = 1$. As β decreases (i.e., crime rates respond more strongly to the allocation), γ decreases, making the optimal interior allocation less sensitive to changes in r_t and r_p^0.

Minimizing Crime Once we entertain the possibility that policing a group more decreases their crime rate, a natural alternative utility function for the officer is that they want to minimize the amount of crime committed. Formally, suppose the officer has a utility $u(p_A, p_B)$ which is decreasing in both arguments, and $p_J(w_J)$ is decreasing in w_J. A simple functional form to use here is:

$$
u(p_A, p_B) = -(t_A p_A)^\alpha - (t_B p_B)^\alpha
$$

where $t_J > 0$ represents how much the officer cares about decreasing crime among each group and $0 < \alpha \leq 1$ represents diminishing returns to reducing crime among each group. Using the same functional from for p_J as above, $p_J(w_J) = p_J^0 w_J^\beta$, gives:

$$
u(w_A) = -(t_A p_A^0)^\alpha w_A^{\alpha\beta} - (t_B p_B^0)^\alpha (1 - w_A)^{\alpha\beta}
$$

Since $-1 < \alpha\beta < 0$, this is concave in w_A, and when the maximizer is interior it lies at the solution to $u'(w_A) = 0$, or:

$$(t_A p_A^0)^\alpha w_A^{\alpha\beta-1} = (t_B p_B^0)^\alpha (1 - w_A)^{\alpha\beta-1} \tag{17}$$

$$\left(\frac{w_A}{1 - w_A}\right)^{\alpha\beta-1} = (r_t r_p^0)^{-\alpha} \tag{18}$$

$$w_A = \frac{(r_t r_p^0)^\delta}{1 + (r_t r_p^0)^\delta} \tag{19}$$

where $\delta = \frac{\alpha}{1-\alpha\beta} > 0$. So the best response can again be expressed as a function of r_t and r_p^0, and is strictly increasing in both ratios.

G Nonlinear Returns to Policing

Returning to the original utility function, recall an additional way to motivate the diminishing returns assumption is that the marginal rate of crimes caught among group J decreases as w_J increases. Suppose the number of crimes caught is equal to $c_J = f(p_J w_J)$ where f is an increasing and concave function. Assume that the officers knows this functional form, but not the p_J parameters.

Knowing c_J and w_J, a fully Bayesian officer could then infer p_J by inverting the f function: $p_J = f^{-1}(c_J)/w_J$. The officer would then form a correct inference about the relative crime "rates" of the group, where the scare quotes highlight that the p_J parameters no long have a simple interpretation as the average crime rates of the groups:

$$\tilde{r}_p(0) = \frac{f^{-1}(c_A)/w_A}{f^{-1}(c_B)/w_B} = p_A/p_B$$

Note that if the officer now beliefs the relative crime rates are equal to c_A/c_B, he is making two mistakes: not adjusting for w_J, and also not accounting for the nonlinear effect of policing effort. In this case his belief about the relative prevalence of crime among members of each group (as a function of the allocation decision) becomes:

$$\frac{f(p_A w_A)}{f(p_B(w - w_A))}$$

Which, as long as f is increasing, is increasing in w_A. Unfortunately with this notion of naivety there is not a natural way to come up with an "intermediate" form of the bias.

One potentially instructive special case is if f is a power function: $f(p_A w_A) = (p_A w_A)^\alpha$,

$\alpha \in (0, 1)$. In this case the fully naive belief simplifies to:

$$\frac{(p_A w_A)^\alpha}{(p_B(w - w_A))^\alpha} = r_p^\alpha \left(\frac{w}{w - w_A} \right)^\alpha$$

If $r_p = 1$ this belief will be correct when $w_A = 1/2$ (and, so with no animus, the officer will again pick a correct allocation). Now when $r_p > 1$, $1 < r_p^\alpha < r_p$. So, if the officer were to allocate his time evenly between the groups, he would now *underestimate* the relative prevalence of crime among members of the group with the higher crime rate. In other words, "not understanding diminishing returns" could lead to the opposite effect as the bias we study.

Another way to model a naive officer is that he is able to "invert" the f function but does not account for the differential policing rate. Such an officer's belief becomes:

$$\frac{p_A w_A}{p_B(w - w_A)}$$

as in the baseline, so we can again define the intermediate form of naivety identically.

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