Supporting Information

"Going Into Government: How Hiring from Special Interests Reduces Their Influence"

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Table of Contents

A Formal Results

The set up of the model is as in the main text of the paper. We restate several important components here.

There are three players, P, G and L, each of whom has the following utility function (over policy location and quality):

$$u_i = b_i q - c_i q_i - \lambda_i(|\hat{x}_i - x|)$$

We make the following assumptions on these utility functions.

Policy loss function. We assume that $-\lambda_i(|\hat{x}_i - x|)$ is single-peaked and symmetric around $\hat{x}_i$. Moreover, we assume that $\lambda_i(0) = \lambda_i'(0) = 0$ and $\lambda_i(y), \lambda_i'(y), \lambda_i'''(y) > 0$ for all $y > 0$. Without loss of generality, we assume that $\hat{x}_P < \hat{x}_G$.

Preferences over quality. Quality is commonly valued among the players. More specifically, we assume that each player i gets a marginal benefit $b_i > 0$ from quality and pays a marginal cost $c_i > 0$ to produce it. We assume further that $b_i < c_i$ (we discuss this in the main text).

For convenience, we will assume that L does not enter government government when she is indifferent.

Public provision

If there is public provision, P may choose to retain the status quo, $x_P = x_0$ and $q_P = q_0 = 0$, or to change it to $x_P \neq x_0$ and $q_G = \overline{q}$.[18] However, $\overline{q}$ is costly. If P decides to change the status quo, it is optimal to set $x_P = \hat{x}_P$. This is optimal if and only if

$$(b_P - c_P)\overline{q} \geq -\lambda_P(|\hat{x}_P - x_0|) \iff c_P \leq b_P + \frac{\lambda_P(|\hat{x}_P - x_0|)}{\overline{q}} \equiv \overline{c}_P$$

Public provision takes the following form. If $c_P > \overline{c}_P$, then $x_P = x_0$ and $q_P = 0$, and if $c_P \leq \overline{c}_P$, then $x_P = \hat{x}_P$ and $q_P = \overline{q}$.

18. In the main text, we provide an interpretation for the requirement that P can only change the status quo if it provides a level of quality $\overline{q}$. For example, P may need to satisfy a veto player or statutory requirements.

Private provision

Now we consider private provision of the service by G. First, we suppose G is willing to provide the service and formally characterize P's optimal private provision.

Equilibrium quality

In order to provide the service, G must induce the government to allow for private provision.

Lemma A1. The special interest group's optimal choice of quality is:

$$q_G^*(x_G) = \max\left\{0, \frac{1}{b_P}\left[\lambda_P(|\hat{x}_P - x_G|) - \psi\right]\right\}$$

where ψ is defined in the proof.

Proof. There are two cases.

Case H: This is the "high capacity" case where $c_P \leq \overline{c}_P$. G must offer a proposal (x_G, q_G) satisfying

$$b_P q_G - \lambda_P(|\hat{x}_P - x_G|) \geq (b_P - c_P)\overline{q} \iff q_G \geq \frac{1}{b_P}\left[\lambda_P(|\hat{x}_P - x_G|) - \overline{q}(c_P - b_P)\right]$$

We use the superscript ^{H} to denote values that correspond to this case.

Case L This is the "low capacity" case where $c_P > \overline{c}_P$. G must offer a proposal (x_G, q_G) satisfying

$$b_P q_G - \lambda_P(|\hat{x}_P - x_G|) \geq -\lambda_P(|\hat{x}_P - x_0|) \iff q_G \geq \frac{1}{b_P}\left[\lambda_P(|\hat{x}_P - x_G|) - \lambda_P(|\hat{x}_P - x_0|)\right]$$

We use the superscript ^{L} to denote values that correspond to this case.

Define the following, which is constant in for all x_G, q_G:

$$\psi = \begin{cases} \overline{q}(c_P - b_P) & \text{if } c_P \leq \overline{c}_P \\ \lambda_P(|\hat{x}_P - x_0|) & \text{if } c_P > \overline{c}_P \end{cases}.$$

Then, the inequalities above can be rewritten as

$$q_G \geq \frac{1}{b_P}\left[\lambda_P(|\hat{x}_P - x_G|) - \psi\right] \tag{3}$$

Since $c_G > b_G$, G optimally sets q_G so that either (3) binds or $q \geq 0$ binds. We can now write G's optimal choice of quality as a function of its choice of the ideological location, x_G:

$$q_G^*(x_G) = \max\left\{0, \frac{1}{b_P}\left[\lambda_P(|\hat{x}_P - x_G|) - \psi\right]\right\} \tag{4}$$

This completes the proof. □

Equilibrium location

Denote G's optimal choice of policy location as $x_G^*(q_G)$. The following interim result allows us to bound the scope of our analysis.

Lemma A2. Suppose G provides the service with quality $q_G^*(x_G)$. Then, $x_G^*(q_G^*(x_G^*)) \in [\hat{x}_P, \hat{x}_G]$.

Proof of Lemma A2. Assume G provides the service with quality $q_G^*(x_G)$. We now prove the result by contradiction. Suppose that $x_G^* < \hat{x}_P$ or $x_G^* > \hat{x}_G$. First, if $x_G^* < \hat{x}_P$, then G has a profitable deviation to $x_G^* = \hat{x}_P$ since $\hat{x}_P$ is closer to $\hat{x}_G$ than x_G^* and $q_G^*(\hat{x}_P) = 0 < q_G^*(x_G^*)$. This contradicts the assumption that x_G^* is a best response. Second, if $x_G^* > \hat{x}_G$, then G has a profitable deviation to $x_G^* = \hat{x}_G$ since $\hat{x}_G$ is closer to $\hat{x}_G$ than x_G^* and $0 \leq q_G^*(\hat{x}_G) < q_G^*(x_G^*)$. This contradicts the assumption that x_G^* is a best response. Thus, since $x_G^*(q_G^*(x_G)) \in \mathbb{R}$, it follows that $x_G^*(q_G^*(x_G)) \in [\hat{x}_P, \hat{x}_G]$. □

Lemma A3. If $\hat{x}_G \leq \hat{x}_G^{\text{corn}}$, then there is a unique $x_G^* = x_G^C$, which satisfies $\lambda_P(|\hat{x}_P - x_G^C|) = \psi$. If $\hat{x}_G > \hat{x}_G^{\text{corn}}$, then there is a unique $x_G^* = x_G^I$, which satisfies $(c_G - b_G)/b_P = \lambda'_G(\hat{x}_G - x_G^I)/\lambda'_P(x_G^I - \hat{x}_P)$.

Proof. Using Lemma A2, we can bound $x_G \in [\hat{x}_P, \hat{x}_G]$. Then, G optimally sets x_G by maximizing

$$\max_{x_G \in [\hat{x}_P, \hat{x}_G]} (b_G - c_G) q_G^*(x_G) - \lambda_G(|\hat{x}_G - x_G|)$$

Interior solutions. Assume there is an interior solution. Then substituting and rearranging, G's problem is to maximize

$$-\left[\frac{c_G - b_G}{b_P}\right](\lambda_P(|\hat{x}_P - x_G|) - \psi) - \lambda_G(|\hat{x}_G - x_G|)$$

Using Lemma A2, and the fact that ψ does not depend on x_G, the maximization problem can be simplified further to:

$$-\left[\frac{c_G - b_G}{b_P}\right]\lambda_P(x_G - \hat{x}_P) - \lambda_G(\hat{x}_G - x_G)$$

The first order condition implicitly defines an optimal, interior policy location, x_G^I:

$$\frac{c_G - b_G}{b_P} = \frac{\lambda'_G(\hat{x}_G - x_G^I)}{\lambda'_P(x_G^I - \hat{x}_P)} \tag{5}$$

Since λ_i is continuous and $\lambda'_i(0) = 0$ and $\lambda'_i(y) > 0$ and $\lambda''_i(y) > 0$ for $y > 0$, note that the right hand side is strictly decreasing x_G^I on the interval $(\hat{x}_P, \hat{x}_G]$ with

$$\lim_{x \to (\hat{x}_P)^+} \frac{\lambda'_G(\hat{x}_G - x)}{\lambda'_P(x - \hat{x}_P)} = \infty \qquad\qquad \frac{\lambda'_G(\hat{x}_G - \hat{x}_G)}{\lambda'_P(\hat{x}_G - \hat{x}_P)} = 0$$

Then, since $(c_G - b_G)/b_P > 0$, there is a unique $x_G^I \in (\hat{x}_P, \hat{x}_G)$ satisfying (5).

Corner solutions. The analysis above presumes that there is an interior solution, meaning that $q_G > 0$. However, a corner solution arises if $q_G = 0$. Using condition (3), $q_G > 0$ requires that

$$\lambda_P(|\hat{x}_P - x_G|) > \psi$$

By the properties of λ_P (i.e., single-peakedness around $\hat{x}_P$), we can define an interval $[\underline{x}_G, \overline{x}_G]$, where $\underline{x}_G$ and $\overline{x}_G$ are roots of:

$$\lambda_P(|\hat{x}_P - x_G|) = \psi$$

Note also that $\underline{x}_G < \hat{x}_P < \overline{x}_G$. Then, if $x_G^I \in [\underline{x}_G, \overline{x}_G]$, an interior solution requires $q_G \leq 0$, so a corner solution must exist. Within this interval, G can get any policy location it wants with no investment in quality. Then, given that $\hat{x}_P < \hat{x}_G$, a corner solution involves $x_G^C = \min\{\hat{x}_G, \overline{x}_G\}$.

Finally, we characterize exactly when a corner solution will arise. Using the analysis above, a corner solution requires

$$\frac{c_G - b_G}{b_P} \geq \frac{\lambda'_G(\hat{x}_G - \overline{x}_G)}{\lambda'_P(\overline{x}_G - \hat{x}_P)} \tag{6}$$

Then, there is a unique $\hat{x}_G$ such that this condition holds with equality. We label this cutoff $\hat{x}_G^{\text{corn}}$ and note that for all $\hat{x}_G > \hat{x}_G^{\text{corn}}$, there is an interior solution and for all $\hat{x}_G \leq \hat{x}_G^{\text{corn}}$ there is a corner solution. □

Condition for private participation

The foregoing analysis assumes that G provides the service. However, G may be better off with public provision by P than with its own private provision. We now analyze G's participation constraint.

Lemma A4. G provides the service if $c_P > \overline{c}_P$ or if $c_P \leq \overline{c}_P$ and $\hat{x}_G \geq \hat{x}_G^{\text{shut}}$.

Proof of Lemma A4. Consider the two cases defined by c_P, as above.

Case L: G's participation constraint is

$$\underbrace{-(c_G - b_G)q_G^*(x_G^*) - \lambda_G(|\hat{x}_G - x_G^*|)}_{u_G(x_G^*, q_G^*)} \geq \underbrace{-\lambda_G(|\hat{x}_G - x_0|)}_{u_G(x_0, 0)} \tag{PC^L}$$

Before proceeding, note that in this case, G always weakly prefers private provision (i.e., participation). We consider corner and interior solutions separately. First, if there's a corner solution then (PC^L) collapses to

$$-\lambda_G(|\hat{x}_G - x_G^C|) \geq -\lambda_G(|\hat{x}_G - x_0|)$$

Note that for a corner solution, it must be that $\lambda_P(|\hat{x}_P - x_G^C|) = \psi = \lambda_P(|\hat{x}_P - x_0|)$. Given that $\hat{x}_P < \hat{x}_G$ and by Lemma A2, it follows that $x_G^C = \max\{x_0, 2\hat{x}_P - x_0, \hat{x}_G\}$. Then $-\lambda_G(|\hat{x}_G - x_G^C|) \geq -\lambda_G(|\hat{x}_G - x_0|)$, as required.

Second, if there's an interior solution, it must be that $\lambda_P(|\hat{x}_P - x_G^I|) > \lambda_P(|\hat{x}_P - x_0|)$. And, given that $\hat{x}_P < \hat{x}_G$ and by Lemma A2, it follows that $x_0 < x_G^I \leq \hat{x}_G$. Since this is an interior solution, G finds it optimal to provide $q_G > 0$ in exchange for x_G^I. Moreover, it could set $x_G^I = x_0$ with $q_G = 0$ but does not find it optimal. Then (PC^L) holds, as required. Therefore, (PC^L) is always satisfied.

Case H: G's participation constraint is

$$\underbrace{-(c_G - b_G)q_G^*(x_G^*) - \lambda_G(|\hat{x}_G - x_G^*|)}_{u_G(x_G^*, q_G^*)} \geq \underbrace{b_G\overline{q} - \lambda_G(|\hat{x}_G - \hat{x}_P|)}_{u_G(\hat{x}_P, \overline{q})} \qquad (PC^H)$$

This condition may or may not hold. First suppose a corner solution. Then:

$$\lambda_G(|\hat{x}_G - \hat{x}_P|) - \lambda_G(|\hat{x}_G - x_G^C|) \geq b_G\overline{q}$$

The LHS is positive by Lemma A2, and the RHS is positive. Moreover, the LHS is strictly increasing in $\hat{x}_G$. Then, if $\hat{x}_G$ is sufficiently high, then the condition holds.

Second, consider an interior solution. Then:

$$-(c_G - b_G)q_G^*(x_G^I) - \lambda_G(|\hat{x}_G - x_G^I|) \geq b_G\overline{q} - \lambda_G(|\hat{x}_G - \hat{x}_P|)$$

We rearrange as follows:

$$\lambda_G(|\hat{x}_G - \hat{x}_P|) - \lambda_G(|\hat{x}_G - x_G^I|) - (c_G - b_G)q_G^*(x_G^I) \geq b_G\overline{q}$$

The LHS is again increasing in $\hat{x}_G$. Then, if $\hat{x}_G$ is sufficiently high, then the condition holds.

Then, a necessary condition for participation is that $\hat{x}_G$ be sufficiently high. Let $\hat{x}_G^{\text{shut}}$ denote the minimum level of $\hat{x}_G$ such that (PC^H) binds. Then $\hat{x}_G \geq \hat{x}_G^{\text{shut}}$ implies PC^H holds. □

Exogenous Entry

Suppose that L goes into government, which increases G's cost to $c_G + \delta_G$ and decreases P's cost to $c_P - \delta_P$. There are three cases to consider: (1) $c_P - \delta_P < c_P \leq \overline{c}_P$, (2) $\overline{c}_P < c_P - \delta_P < c_P$, and (3) $c_P - \delta_P \leq \overline{c}_P < c_P$.

Let $\tilde{\delta}$ be defined by $\tilde{\delta} \equiv \max\{0, c_P - \overline{c}_P\}$ and let $\overline{\delta}$ be defined by $\overline{\delta} = c_P - b_P$. We can rewrite these cases in terms of δ_P:

- **Case HH:** If $\tilde{\delta} = 0$, then P has high capacity with or without entry.
- **Case LL:** If $0 < \delta_P < \tilde{\delta}$, then P has low capacity with or without entry.
- **Case LH:** If $0 < \tilde{\delta} \leq \delta_P$, then P has low capacity without entry and high capacity with entry.

In the following, we use the following notation. Let $x_G^\delta \equiv x_G^*(c_P - \delta_P, c_G + \delta_G), q_G^\delta \equiv q_G^*(c_P - \delta_P, c_G + \delta_G)$ and $x_G^0 \equiv x_G^*(c_P, c_G), q_G^0 \equiv q_G^*(c_P, c_G)$

> **Lemma A5.** Private provision occurs weakly less often after the lobbyist enters government, and strictly less often if entry dramatically increases the policymaker's capacity. Formally, if $\delta_P < \tilde{\delta}$, entry has no effect on whether private provision occurs, and if $\delta_P \geq \tilde{\delta}$, then there exist some $\hat{x}_G$ such that private provision occurs without entry, but not with entry.

Proof. We consider three cases.

Case LL. Suppose $0 < \delta_P < \tilde{\delta}$. In this case, (PC^L) is G's participation constraint before and after L's entry. Since (PC^L) is always satisfied (by analysis above), a useful corollary is that if $\delta_P < \tilde{\delta}$, then G participates before and after entry.

Case HH. Suppose $\tilde{\delta} = 0$. In this case, (PC^H) is G's participation constraint before and after L's entry. Using the envelope theorem, the left-hand side of (PC^H) is decreasing in c_G and increasing in c_P. Then, the participation constraint is always harder to satisfy after entry, implying that $\hat{x}_G^{\text{shut}}$ is higher after entry.

Case LH. Suppose $0 < \tilde{\delta} \leq \delta_P$. In this case, L's entry enables P to publicly provide the service. Since (PC^L) always holds, then (PC^H) holds after entry for all $\hat{x}_G \geq \hat{x}_G^{\text{shut}}$. □

Next we consider the effect of entry on private provision, assuming participation constraint holds before and after. Before proceeding, note that $\hat{x}_G^{\text{corn}}$ depends on c_G and c_P, and will therefore depend on whether L enters. Denote the no entry and entry cutoffs as $\hat{x}_G^{\text{corn}}(0)$ and $\hat{x}_G^{\text{corn}}(\delta)$, respectively.

> **Lemma A6.** The ideological location of policy shifts toward the policymaker after entry. Formally, $\hat{x}_P \leq x_G^\delta \leq x_G^0 \leq \hat{x}_G$, where $x_G^\delta < x_G^0$ if $c_P - \delta_P \leq \overline{c}_P$ or $\hat{x}_G > \min\{\hat{x}_G^{\text{corn}}(0), \hat{x}_G^{\text{corn}}(\delta)\}$.

Proof. First note that $\overline{x}_G$ weakly decreases after entry. Label the before- and after-entry corner solutions as $\overline{x}_G(0)$ and $\overline{x}_G(\delta)$, respectively. We now consider three cases.

Case 1. Suppose $\hat{x}_G \leq \min\{\hat{x}_G^{\text{corn}}(0), \hat{x}_G^{\text{corn}}(\delta)\}$. Then $x_G^\delta < x_G^0$ if $\overline{x}_G(\delta) < \overline{x}_G(0)$ and $x_G^\delta = x_G^0$ $\overline{x}_G(\delta) = \overline{x}_G(0)$. Moreover, the latter case only arises if $c_P - \delta_P > \overline{c}_P$.

Case 2. Suppose $\hat{x}_G > \max\{\hat{x}_G^{\text{corn}}(0), \hat{x}_G^{\text{corn}}(\delta)\}$. Then from (5), it is straight forward to see that $x_G^\delta < x_G^0$.

Case 3. Suppose $\hat{x}_G \in (\hat{x}_G^{\text{corn}}(0), \hat{x}_G^{\text{corn}}(\delta)]$. Then there is an interior solution before entry and a corner solution after entry. If $c_P - \delta_P > \bar{c}_P$ (Case LL), then $\overline{x}_G(0) = \overline{x}_G(\delta)$, which implies $x_G^\delta < x_G^0$ since x_G^0 is interior. If $c_P \leq \bar{c}_P$ (Case HH), then since $\overline{x}_G(\delta) < \overline{x}_G(0)$ and x_G^0 is interior (and thus greater than $\overline{x}_G(0)$), it follows that $x_G^\delta < x_G^0$. If $c_P - \delta_P \leq \bar{c}_P < c_P$ (Case LH), then $x_G^\delta < x_G^0$ since $c_P - \delta_P \leq \bar{c}_P$.

Case 4. Suppose $\hat{x}_G \in (\hat{x}_G^{\text{corn}}(\delta), \hat{x}_G^{\text{corn}}(0)]$. Then there is a corner solution before entry and an interior solution after entry. Assume by contradiction that $x_G^\delta \geq x_G^0 = \overline{x}_G(0)$. Then:

$$x_G^\delta \geq \overline{x}_G(0) \Rightarrow \frac{\lambda_G'(\hat{x}_G - x_G^\delta)}{\lambda_P'(x_G^\delta - \hat{x}_P)} \leq \frac{\lambda_G'(\hat{x}_G - \overline{x}_G(0))}{\lambda_P'(\overline{x}_G(0) - \hat{x}_P)}$$

And since x_G^δ is interior and $\overline{x}_G(0)$ is corner, then:

$$\frac{c_G + \delta_G - b_G}{b_P} = \frac{\lambda_G'(\hat{x}_G - x_G^\delta)}{\lambda_P'(x_G^\delta - \hat{x}_P)} \leq \frac{\lambda_G'(\hat{x}_G - \overline{x}_G(0))}{\lambda_P'(\overline{x}_G(0) - \hat{x}_P)} \leq \frac{c_G - b_G}{b_P}$$

But since $\delta_G > 0$, this is a contradiction since the first term is strictly larger than the last term. Then $x_G^\delta < x_G^0$. Finally, by Lemma A2, $\hat{x}_P \leq x_G^\delta \leq x_G^0 \leq \hat{x}_G$ □

Endogenous Entry

Because we assume L does not enter when indifferent, endogenous entry by L requires:

$$b_L q^\delta - \lambda_G(|\hat{x}_G - x^\delta|) > b_L q^0 - \lambda_G(|\hat{x}_G - x^0|)$$

Rearranging yields:

$$b_L(q^\delta - q^0) > \lambda_G(|\hat{x}_G - x^\delta|) - \lambda_G(|\hat{x}_G - x^0|)$$

First note that if there is no private provision with or without entry, then L prefers to enter only if $\delta_P \geq \tilde{\delta} > 0$ so that entry improves the policymaker's capacity.

Now we consider the cases in which private provision occurs without entry. It is immediate to see that if $q^\delta = 0$, this will automatically fail since $\hat{x}_P < x_G^\delta \leq x_G^0 < \hat{x}_G$ and the right hand side is weakly positive.

Next suppose $q^\delta > 0$. There are two situations to consider. First, $q^\delta = q_P = \overline{q}$.

Lemma A7. Suppose G's participation constraint is satisfied if $e = 0$ and fails if $e = 1$, and that $q^\delta = \overline{q}$. Then L will optimally set $e = 0$, unless

$\delta_P \geq \tilde{\delta} > 0$ and $\overline{q} > \tilde{q}(\hat{x}_G)$, where $\tilde{q}(\hat{x}_G)$ is defined in the proof.

Proof of Lemma A7. Suppose by contradiction, that G's participation constraint fails with $e = 1$ and L sets $e = 1$. For $e = 1$ to be optimal, it must follow that:

$$b_L q_P^\delta - \lambda_G(|\hat{x}_G - x_P^\delta|) > b_L q_G^0 - \lambda_G(|\hat{x}_G - x_G^0|) \tag{7}$$

Since G's participation constraint is satisfied if $e = 0$ but fails if $e = 1$, it follows that either $\tilde{\delta} = 0$ (Case HH) or $\delta_P \geq \tilde{\delta} > 0$ (Case LH). Now, consider each case.

Case HH. This implies that $q_P^0 = q_P^\delta = \overline{q}$ and $x_P^0 = x_P^\delta = \hat{x}_P$. Since G finds participation optimal before entry, it must be that $b_L q_G^0 - \lambda_G(|\hat{x}_G - x_G^0|) > b_L q_P^0 - \lambda_G(|\hat{x}_G - x_P^0|)$. Then using (7),

$$b_L(\overline{q} - q_P^0) > \lambda_G(|\hat{x}_G - \hat{x}_P|) - \lambda_G(|\hat{x}_G - x_P^0|)$$

Both sides are zero and the condition fails, a contradiction.

Case LH. This implies that $q_P^\delta = \overline{q}$ and $x_P^\delta = \hat{x}_P$, and that $q_P^0 = 0$ and $x_P^0 = x_0$. First, suppose there's a corner solution so that $q_G^0 = 0$. Then (7) collapses to

$$\overline{q} > \frac{1}{b_L}\left[\lambda_G(|\hat{x}_G - \hat{x}_P|) - \lambda_G(|\hat{x}_G - x_G^0|)\right]$$

Second, suppose there's an interior solution so that $q_G^0 > 0$. Then

$$q_G^0 = \frac{1}{b_P}[\lambda_P(|\hat{x}_P - x_G^0|) - \lambda_P(|\hat{x}_P - x_0|)]$$

And (7) collapses to

$$\overline{q} > \frac{1}{b_P}\left[\lambda_P(|\hat{x}_P - x_G^0|) - \lambda_P(|\hat{x}_P - x_0|)\right] + \frac{1}{b_L}\left[\lambda_G(|\hat{x}_G - \hat{x}_P|) - \lambda_G(|\hat{x}_G - x_G^0|)\right]$$

Define

$$\tilde{q}(\hat{x}_G) \equiv \begin{cases} \dfrac{\lambda_P(|\hat{x}_P - x_G^C|) - \lambda_P(|\hat{x}_P - x_0|)}{b_P} & \text{if } \hat{x}_G \leq \hat{x}_G^{\text{corn}} \\ \dfrac{\lambda_P(|\hat{x}_P - x_G^I|) - \lambda_P(|\hat{x}_P - x_0|)}{b_P} + \dfrac{\lambda_G(|\hat{x}_G - \hat{x}_P|) - \lambda_G(|\hat{x}_G - x_G^I|)}{b_L} & \text{if } \hat{x}_G > \hat{x}_G^{\text{corn}} \end{cases}$$

Then, if $\overline{q} > \tilde{q}(\hat{x}_G)$, then (7) is satisfied and $e = 1$. Otherwise $e = 0$. $\square$

Second, $q^\delta = q_G^* > 0$.

Lemma A8. Suppose G's participation constraint is satisfied if $e = 0$ and if $e = 1$, and that $q^\delta = q^\delta_G > 0$. Then L will optimally set $e = 1$, if δ_P is sufficiently high and δ_G is sufficiently low.

Proof of Lemma A7. **Case LL:** Suppose that L enters. Since the condition for a corner solution is the same regardless of entry, there is interior effort with or without entry. Then:

$$\frac{1}{b_P}\left[\lambda_P(|\hat{x}_P - x^\delta_G|) - \lambda_P(|\hat{x}_P - x^0_G|)\right] > \lambda_G(|\hat{x}_G - x^\delta_G|) - \lambda_G(|\hat{x}_G - x^0_G|)$$

By Lemma A6, $\hat{x}_P < x^\delta_G < x^0_G < \hat{x}_G$. Using the properties of λ_i, the LHS is strictly negative and the RHS is strictly positive, a contradiction. Then if $\delta_P < \tilde{\delta}$, there is no endogenous entry.

Case HH: Suppose that L enters. Then

$$\frac{1}{b_P}[\lambda_P(|\hat{x}_P - x^\delta_G|) - \overline{q}(c_P - b_P - \delta_P)] - \lambda_G(|\hat{x}_G - x^\delta_G|) > q^0_G - \lambda_G(|\hat{x}_G - x^0_G|)$$

Rearranging, this becomes

$$[\lambda_P(|\hat{x}_P - x^\delta_G|) - \overline{q}(c_P - b_P - \delta_P)] - b_P q^0_G > b_P\left[\lambda_G(|\hat{x}_G - x^\delta_G|) - \lambda_G(|\hat{x}_G - x^0_G|)\right]$$

With an interior solution,

$$\overline{q}\delta_P > \lambda_P(|\hat{x}_P - x^0_G|) - \lambda_P(|\hat{x}_P - x^\delta_G|) + b_P\left[\lambda_G(|\hat{x}_G - x^\delta_G|) - \lambda_G(|\hat{x}_G - x^0_G|)\right]$$

With a corner solution,

$$\overline{q}\delta_P - \overline{q}(c_P - b_P) > b_P\left[\lambda_G(|\hat{x}_G - x^\delta_G|) - \lambda_G(|\hat{x}_G - x^0_G|)\right] - \lambda_P(|\hat{x}_P - x^\delta_G|)$$

Note again that $\hat{x}_P < x^\delta_G < x^0_G < \hat{x}_G$. So, by the properties of λ_i, RHS is positive for both conditions.

Case LH. If L enters, then P is high capacity, but if she does not enter, then P is low capacity. Entry is a best response if and only if

$$\frac{1}{b_P}[\lambda_P(|\hat{x}_P - x^\delta_G|) - \overline{q}(c_P - \delta_P - b_P)] - \lambda_G(|\hat{x}_G - x^\delta_G|) > q^0_G - \lambda_G(|\hat{x}_G - x^0_G|)$$

With an interior solution,

$$\overline{q}\delta_P - \overline{q}(c_P - b_P) + \lambda_P(|\hat{x}_P - x_0|) > \lambda_P(|\hat{x}_P - x_G^0|) - \lambda_P(|\hat{x}_P - x_G^\delta|) + b_P\left[\lambda_G(|\hat{x}_G - x_G^\delta|) - \lambda_G(|\hat{x}_G - x_G^0|)\right]$$

With a corner solution,

$$\overline{q}\delta_P - \overline{q}(c_P - b_P) > b_P[\lambda_G(|\hat{x}_G - x_G^\delta|) - \lambda_G(|\hat{x}_G - x_G^0|)] - \lambda_P(|\hat{x}_P - x_G^\delta|)$$

Finally, note that for each of these conditions, the LHS increases in δ_P, while the RHS increases in δ_G. Moreover, each holds for $\delta_P = \overline{\delta}_P$ and $\delta_G = 0$. Then, entry occurs if δ_P is sufficiently high and δ_G is sufficiently low. □