Technical Appendix:
"Getting a Fair Hearing? Bias and Error in the Judicial Hierarchy"

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July 12, 2018

FOR ONLINE PUBLICATION

Assumptions

Assumption 1 (no fabrication)**.** The case merits cannot be fabricated.

Assumption 2 (powerful litigant)**.** The defendant is the "more powerful" litigant, so that $c_D < c_P$.

Assumption 3 (sufficient resource constraints)**.** For the litigants, $c_P > c_D > 1$. For the judge, $c_T > \bar{c}_T$, where $\bar{c}_T$ is defined in the Appendix.

Assumption 4 (indifference)**.** When indifferent, the judge rules in favor of the defendant and the appellate court's reversal strategy favors the defendant.

Proofs

Lemma A1. Let $\hat{\pi}_A$ be A's posterior belief that $\omega = P$ at its information set. In equilibrium, A's optimal reversal strategy is

$$r^*(\hat{\pi}_A, x) = \begin{cases} 1 & \text{if } \left[\hat{\pi}_A \leq \dfrac{1}{2} \text{ and } x = P\right] \text{ or } \left[\hat{\pi}_A > \dfrac{1}{2} \text{ and } x = D\right] \\ 0 & \text{otherwise} \end{cases}$$

Proof of Lemma A1. This is direct from the requirement of perfect Bayesian equilibrium that players best respond to their beliefs. Since A prefers $x = \omega$ to $x \neq \omega$, then A reverses if and only if it believes $x \neq \omega$ is more probable than $x = \omega$. Since $\hat{\pi}_A$ is A's equilibrium posterior that $\omega = P$, then it is optimal to reverse if and only if $\left[\hat{\pi}_A \leq \frac{1}{2} \text{ and } x = P\right]$ or $\left[\hat{\pi}_A > \frac{1}{2} \text{ and } x = D\right]$. □

Lemma A2. In equilibrium, $b_L^* = \omega$ if and only if $m_L = \omega \neq x$. Moreover:

$$a_L^*(m_T, x) = \begin{cases} \dfrac{\pi}{c_P} & \text{if } m_T = \phi \text{ and } x = D \\ \dfrac{1-\pi}{c_D} & \text{if } m_T = \phi \text{ and } x = P \\ 0 & \text{if } m_T = \omega \text{ or } r(\hat{\pi}_A, x) = 1 \end{cases} \tag{1}$$

Proof of Lemma A2. First, we consider L's equilibrium brief. Since L pays a cost $\varepsilon_L > 0$ when $b_L \neq L$, it follows that the only candidate brief is $m_L = \omega \neq x$. Now we verify if it is feasible and optimal. First, if $m_L = \phi$, then $m_L = \omega$ is not feasible since L has no information, and by Assumption 1, L cannot fabricate information. Next, if $m_L = \omega = L$, then $x \neq L$ and by Lemma A1, A reverses x. This makes L strictly better off. Therefore, $b_L^* = \omega$ if and only if $m_L = \omega \neq x$.

We next consider L's equilibrium appeal effort. It is straightforward to see that if $r(\hat{\pi}_A, x) = 1$, then L secures reversal regardless of her brief and makes no costly effort to discover an error. By Lemma A1 and given b_L^*, L's ex ante expected utility is

$$U_L(a_L, m_T) = \begin{cases} a_P\pi - \frac{1}{2}c_P a_P^2 & \text{if } m_T = \phi \text{ and } L = P \\ a_D(1-\pi) - \frac{1}{2}c_D a_D^2 & \text{if } m_T = \phi \text{ and } L = D \\ -\frac{1}{2}c_D a_D^2 & \text{if } m_T = \omega \neq L \\ 1 - \frac{1}{2}c_D a_D^2 & \text{if } m_T = \omega = L \end{cases}$$

Maximizing this with respect to a_L yields

$$a_L^* = \begin{cases} \frac{\pi}{c_P} & \text{if } m_T = \phi \text{ and } x^* = D \\ \frac{1-\pi}{c_D} & \text{if } m_T = \phi \text{ and } x^* = P \\ 0 & \text{if } m_T = \omega \end{cases}$$

Finally, note that the second derivative is negative, guaranteeing a_L^* is a maximum. □

Lemma A3. Let $x_{m_T}^*$ indicate the trial judge's equilibrium judgment, conditional on her information m_T. Then, in equilibrium:

$$x_\omega^* = \omega \qquad\qquad x_\phi^* = \begin{cases} D & \text{if } \pi \leq \min\{\tilde{\pi}_\delta, \overline{\pi}\} \\ P & \text{if } \pi > \min\{\tilde{\pi}_\delta, \overline{\pi}\} \end{cases}$$

where $\tilde{\pi}_\delta$ and $\overline{\pi}$ are defined in the proof.

Proof of Lemma A3. It is straight forward to see that if $m_L = \omega$, she is better with $x_\omega^* = \omega$ than a deviation $x_\omega' \neq \omega$, which is reversed at cost k. Equilibrium reversal by A follows directly from A's preferences and the fact that m_T is public.

Now consider T's optimal decision when uninformed, which depends on A's reversal strategy. First, suppose that A always affirms T's judgment when A is uninformed.

Then T rules for the defendant if and only if

$$\underbrace{(1-\hat{\pi}_T)\delta + \hat{\pi}_T a_P^*(\delta - k)}_{U_T(x=D)} \geq \underbrace{\hat{\pi}_T\delta + (1-\hat{\pi}_T)a_D^*(\delta - k)}_{U_T(x=P)}$$

Using Lemma A2 and the fact that $\hat{\pi}_T = \pi$ when $m_T = \phi$, this reduces to

$$(1-2\pi)\delta + (\delta - k)\left(\frac{\pi^2 c_D - (1-\pi)^2 c_P}{c_D c_P}\right) \geq 0 \tag{2}$$

Using the quadratic formula, the roots of the left hand side are:

$$\Pi = \left\{\frac{c_P(\delta t(c_D - 1) + k) \pm \sqrt{c_D c_P(\delta t(c_D - 1) + k)(\delta t(c_P - 1) + k)}}{(\delta t - k)(c_D - c_P)}\right\} \tag{3}$$

Given that $c_P > c_D$ and $0 \leq \pi \leq 1$ then equation (3) reduces to the following:

$$\tilde{\pi}_\delta = \frac{c_P(\delta(c_D - 1) + k) - \sqrt{c_D c_P(\delta(c_D - 1) + k)(\delta(c_P - 1) + k)}}{(k - \delta)(c_P - c_D)} \tag{4}$$

Now we establish that $\tilde{\pi}_\delta$ characterizes a cutpoint such that (2) holds if and only if $\pi \leq \tilde{\pi}_\delta$. This follows from these facts:

1. (2) is decreasing in π since its first derivative is negative:

$$-2\delta\left(1 - \frac{c_P - \pi(c_P - c_D)}{c_D c_P}\right) - \frac{2k(c_P - \pi(c_P - c_D))}{c_D c_P} < 0$$

2. the left hand side is strictly positive at $\pi = 0$ and strictly negative at $\pi = 1$

Next, we verify that it is sequentially rational for A to affirm T's decision when uninformed. Affirming is a best response to the following beliefs:

$$\underbrace{\frac{\pi}{\pi + (1-\pi)(1-a_D^*)}}_{\hat{\pi}_A(x^*=P, m_T=b_L=\phi)} \geq \frac{1}{2} \qquad \underbrace{\frac{\pi(1-a_P^*)}{\pi(1-a_P^*) + 1 - \pi}}_{\hat{\pi}_A(x^*=D, m_T=b_L=\phi)} \leq \frac{1}{2}$$

Substituting and rearranging yields

$$\pi \geq \sqrt{(c_D - 1)c_D} - (c_D - 1) < \frac{1}{2} \equiv \underline{\pi} \qquad \pi \leq c_P - \sqrt{(c_P - 1)c_P} \equiv \overline{\pi}$$

It is straightforward to verify that $\tilde{\pi}_\delta > \underline{\pi}$ for all c_D, c_P. However, it is not true that $\pi_\delta < \overline{\pi}$ for all c_D, c_P. Assuming that T finds it optimal to avoid reversal, then $x_\phi^* = D$ if and only if $\pi \leq \tilde{\pi}_\delta$ and $\pi \leq \overline{\pi}$. Finally, we verify that for all $\pi \in [\overline{\pi}, \tilde{\pi}_\delta]$ (if such an interval exists), it is indeed optimal that T sets $x = P$ in order to avoid reversal. Suppose, by contradiction that there is a profitable deviation to $x = D$. Then, A reverses in the absence of information, and $a_L = 0$. However, since $a_L = 0$ it follows that $\hat{\pi}_A = \tilde{\pi}_\delta = \pi$ and there exists no interval $[\overline{\pi}, \tilde{\pi}_\delta]$. We have now fully established T's optimal judgment. □

Lemma A4. In equilibrium, T exerts effort according to:

$$e^*(x_\phi^*) = \begin{cases} \frac{1}{c_T c_D}\left[(1-\pi)(c_D - (1-\pi))\delta + (1-\pi)^2 k\right] & \text{if } x_\phi^* = P \\ \frac{1}{c_T c_P}\left[\pi(c_P - \pi)\delta + \pi^2 k\right] & \text{if } x_\phi^* = D \end{cases} \tag{5}$$

Proof of Lemma A4. In text. □

Proposition A1. There exists a perfect Bayesian equilibrium of the game.

Proof of Proposition A1. A perfect Bayesian equilibrium is a sequentially rational strategy profile and consistent beliefs. Existence of a sequentially rational profile follows directly from Lemmas A1 to A4, which characterize the player's equilibrium strategies. Relevant beliefs are over the merits, ω. Whenever ω is revealed, all player's beliefs become degenerate and subsequent subgames no longer feature imperfect information. The equilibrium beliefs of T and A are straight forward due to the fact that T's information is public. Specifically, they collapse to either the prior π or to certainty. A's belief when $s_T = b_L = \phi$ is more complicated, since L may be concealing information. The proof of Lemma A3 characterizes the equilibrium belief, which is consistent since it is formed by Bayes' rule using the players' equilibrium strategies. □

Proposition A2. If δ is not too high, then a judge is less impartial when reversal costs are higher. Formally, if $\delta < k$, then $\frac{1}{2} \leq \tilde{\pi}_\delta(k) < \tilde{\pi}_\delta(k')$ for $k' > k$.

Proof of Proposition A2. Suppose that $\delta < k$. First, I show that $\tilde{\pi}_\delta(k) > \frac{1}{2}$ for all $k > \delta$. By contradiction, suppose $\tilde{\pi}_\delta \leq \frac{1}{2}$. Recall definition of $\tilde{\pi}_\delta$,

$$(1 - 2\tilde{\pi}_\delta)\delta + \underbrace{(\delta - k)}_{-}\left(\frac{\tilde{\pi}_\delta^2 c_D - (1 - \tilde{\pi}_\delta)^2 c_P}{c_D c_P}\right) = 0 \tag{6}$$

Since $\tilde{\pi}_\delta \leq \frac{1}{2}$, the first term is weakly positive. Moreover, the third term is strictly negative since:

$$\tilde{\pi}_\delta^2 c_D - (1 - \tilde{\pi}_\delta)^2 c_P \geq 0 \Rightarrow \tilde{\pi}_\delta \geq \frac{c_P}{c_P - c_D} - \sqrt{\frac{c_P c_D}{(c_D - c_P)^2}} > \frac{1}{2}$$

a contradiction. Therefore, if $\tilde{\pi}_\delta < \frac{1}{2}$, the left hand side of Equation (6) is strictly positive, a contradiction. We have shown $\tilde{\pi}_\delta(k) > \frac{1}{2}$ for all k. It remains to be shown that $\tilde{\pi}_\delta(k) < \tilde{\pi}_\delta(k')$ for all $k' > k$. We show this directly using the derivative of $\tilde{\pi}_\delta(k)$:

$$\frac{\partial \tilde{\pi}_\delta(k)}{\partial k} = \frac{\delta c_D c_P \left(k(c_D + c_P) + \delta(2c_D c_P - c_D - c_P) - 2\sqrt{c_D c_P(\delta(c_D - 1) + k)(\delta(c_P - 1) + k)} \right)}{2(\delta - k)^2 (c_P - c_D)\sqrt{c_D c_P(\delta(c_D - 1) + k)(\delta(c_P - 1) + k)}}$$

This is strictly positive since

$$k(c_D + c_P) + \delta(2c_D c_P - c_D - c_P) - 2\sqrt{c_D c_P(\delta(c_D - 1) + k)(\delta(c_P - 1) + k)} > 0$$

Therefore, $\frac{\partial \tilde{\pi}_\delta(k)}{\partial k} > 0$, implying $\tilde{\pi}_\delta(k) < \tilde{\pi}_\delta(k')$ for all $k' > k$. Finally, since $\tilde{\pi}_\delta(k) > \frac{1}{2}$ for all $k > \delta$, we have proven that $\frac{1}{2} < \tilde{\pi}_\delta(k) < \tilde{\pi}_\delta(k')$. □

Proposition A3. Suppose that the systemic bias in favor of the powerful litigant is not too great. Then the trial judge's optimal judgment minimizes judicial effort. Formally, suppose $\tilde{\pi}_\delta \leq \overline{\pi}$. Then, $e^*(x^*_\phi = D) \leq e^*(x^*_\phi = P)$ if and only if $x^*_\phi = D$.

Proof of Proposition A3. We show this directly:

$$\begin{aligned} & e^*(x^*_\phi = P) \geq e^*(x^*_\phi = D) \\ & \iff \frac{1}{c_T c_D}\left[(1-\pi)(c_D - (1-\pi))\delta + (1-\pi)^2 k\right] \geq \frac{1}{c_T c_P}\left[\pi(c_P - \pi)\delta + \pi^2 k\right] \\ & \iff \pi \leq \tilde{\pi}_\delta \\ & \iff x^*_\phi = D \end{aligned}$$

The last step comes from Lemma A3. □

Lemma A5. The equilibrium effort of T has the following characteristics:

- e^* is strictly increasing on the interval $[0, \min\{\tilde{\pi}_\delta, \overline{\pi}\})$ and strictly decreasing on the interval $(\min\{\tilde{\pi}_\delta, \overline{\pi}\}, 1]$.
- e^* is strictly increasing in δ and k, and strictly decreasing in c_T.

Proof of Lemma A5. First notice that Lemma A3 establishes that $x^*_\phi = D$ for all $\pi \leq \min\{\tilde{\pi}_\delta, \overline{\pi}\}$ and $x^*_\phi = P$ for all $\pi > \min\{\tilde{\pi}_\delta, \overline{\pi}\}$. Then, first and second derivatives of e^* with respect to π are given by:

$$\frac{\partial e^*}{\partial \pi} = \begin{cases} \dfrac{c_P\delta + 2\pi(k-\delta)}{c_T c_P} \\ -\dfrac{c_D\delta + 2(1-\pi)(k-\delta)}{c_T c_D} \end{cases} \quad \text{and} \quad \frac{\partial (e^*)^2}{\partial^2 \pi} = \begin{cases} \dfrac{2(k-\delta)}{c_T c_P} & \text{if } \pi \leq \min\{\tilde{\pi}_\delta, \overline{\pi}\} \\ \dfrac{2(k-\delta)}{c_T c_D} & \text{if } \pi > \min\{\tilde{\pi}_\delta, \overline{\pi}\} \end{cases}$$

If $t = 0$ or if $k \geq \delta$, then it is apparent from inspection of the first derivative that the equilibrium effort is strictly increasing when $\pi \leq \min\{\tilde{\pi}_\delta, \overline{\pi}\}$ and strictly decreasing when $\pi > \min\{\tilde{\pi}_\delta, \overline{\pi}\}$. If $t = 1$ and $\delta > k$, then to prove the claim, it suffices to show that $c_P\delta + 2\pi(k-\delta) > 0$ for all $\pi \leq \min\{\tilde{\pi}_\delta, \overline{\pi}\}$ and $-c_D\delta - 2(1-\pi)(k-\delta) < 0$ for all $\pi > \min\{\tilde{\pi}_\delta, \overline{\pi}\}$. Thus, the proof proceeds by showing that

$$\tilde{\pi}_\delta < \frac{c_P\delta}{2(\delta-k)} \quad \text{and} \quad \frac{2(\delta-k) - c_D\delta}{2(\delta-k)} < \overline{\pi}$$

Using the $\tilde{\pi}_\delta$ and $\overline{\pi}$ from Lemma A3, it is straight forward to verify that these conditions hold. The final set of comparative statics in the second bullet are easily obtained by inspecting e^*. □

Lemma A6. On an issue i, $e^*(\delta_i = \delta) > e^*(\delta_i = 0)$.

Proof of Lemma A6. To prove this result, I show that the equilibrium effort of a judge with $\delta_i = \delta$ is greater than the equilibrium effort of a judge with $\delta_i = 0$ for all $\pi \in [0, 1]$. Since $c_P > c_D$, there are three cases to consider: $\pi < \tilde{\pi}_\delta$, $\pi \in [\tilde{\pi}_\delta, \tilde{\pi}_0]$, $\pi > \tilde{\pi}_0$. The first and third cases involve both kinds adjudicating in the same way: in favor of the defendant and the plaintiff respectively. Suppose Assumption 3 holds. It is straightforward to see from inspection of equation (5) that a $\delta_i = 0$ judge exerts strictly less effort than a $\delta_i = \delta$ judge when they have the same predisposition.

Now consider the second case, $\pi \in [\tilde{\pi}_\delta, \tilde{\pi}_0]$. By Lemma A5, $e^*(\pi, \delta_i = \delta)$ is strictly decreasing and $e^*(\pi, \delta_i = 0)$ is strictly increasing for all $\pi \in (\tilde{\pi}_\delta, \tilde{\pi}_0)$. Then to show that $e^*(\pi, \delta_i = \delta) > e^*(\pi, \delta_i = 0)$ in this interval, it suffices to show that $e^*(\overline{\pi}, \delta_i = \delta) > e^*(\overline{\pi}, \delta_i = 0)$:

$$\frac{1}{c_T c_D}\left[(1-\overline{\pi})(c_D - (1-\overline{\pi}))\delta t + (1-\overline{\pi})^2 k\right] > \frac{k(1-\overline{\pi})^2}{c_T c_D}$$

The condition holds if $c_D > 1$, thus completing the proof. □

Lemma A7. An issue 1 judge is a lower quality adjudicator on an issue 2 case than an issue 2 judge, and vice versa. Formally, $\xi(x^*_\phi, \delta) > \xi(x^*_\phi, 0)$.

Proof of Lemma A7. Since $c_P > c_D$, there are three cases to consider: (I) $\pi < \tilde{\pi}_\delta$, (II) $\pi > \tilde{\pi}_0$ and (III) $\pi \in [\tilde{\pi}_\delta, \tilde{\pi}_0]$. I show that the *ex ante* probability of a correct decision is greater for a $\delta_i = \delta$ than for a $\delta_i = 0$ in each case.

First, define $\xi(x^*_\phi, \delta_i)$ by substituting in the equilibrium values:

$$\xi(x_\phi = P, \delta_i) = \pi + \frac{(1-\pi)^2}{c_D} + \frac{(1-\pi)^2(c_D - (1-\pi))(\delta t(c_D - (1-\pi)) + k(1-\pi))}{c_D^2 c_T} \quad (7)$$

Similarly,

$$\xi(x_\phi = D, \delta_i) = 1 - \pi + \frac{\pi^2}{c_P} + \frac{\pi^2(c_P - \pi)(\delta t(c_P - \pi) + k\pi)}{c_P^2 c_T} \quad (8)$$

Cases (I) and (II). From equations (7) and (8), it is apparent that $\xi(x^*_\phi, \delta_i = \delta)$ is strictly greater than $\xi(x^*_\phi, \delta_i = 0)$ since $(1-\pi)^2(c_D - (1-\pi)) > 0$ and $\pi^2(c_P - \pi) > 0$.

Case (III). By contradiction, suppose that $\xi(x_\phi = P, \delta)$ were weakly less than $\xi(x_\phi = D, 0)$ at some point in the interval $[\tilde{\pi}_\delta, \tilde{\pi}_0]$. Then, by the definition of $\xi(\cdot)$, this implies the *ex ante* probability that $y^* = \omega$ is lower for the $\delta_i = \delta$ judge than for the $\delta_i = 0$ judge at such a point. Using the fact that, when uninformed, a $\delta_i = 0$ judge rules for the defendant and a $\delta_i = \delta$ judge rules for the plaintiff, we can rewrite the *ex ante* probability as follows:

$$\xi(x^*_\phi, \delta_i) = \begin{cases} e^*(x^*_\phi, \delta_i) + [1 - e^*(x^*_\phi, \delta_i)][1 - \pi(1 - a^*_P)] & \text{if } \delta_i = 0 \\ e^*(x^*_\phi, \delta_i) + [1 - e^*(x^*_\phi, \delta_i)][1 - (1-\pi)(1 - a^*_D)] & \text{if } \delta_i = \delta \end{cases}$$

By Lemma A6, $e^*(x^*_\phi = P, \delta_i = \delta) > e^*(x^*_\phi = D, \delta_i = 0)$. In order for $\xi(x_\phi = P, \delta_i = \delta) \leq \xi(x_\phi = D, \delta_i = 0)$ for some $\pi \in [\tilde{\pi}_1, \tilde{\pi}_0]$ as assumed, it must be that:

$$1 - \pi(1 - a^*_P) > 1 - (1-\pi)(1 - a^*_D)$$

Substituting equilibrium values and simplifying yields:

$$1 > \frac{(1-\pi)^2}{c_D} - \frac{\pi^2}{c_P} + 2\pi$$

The right hand side is increasing in π, and thus is at its smallest when $\tilde{\pi}_\delta$:

$$1 > \frac{\delta^2(c_P - c_D) + \delta k(2c_D c_P + c_D - 3c_P) - 2k\left(\sqrt{c_D c_P(\delta(c_D - 1) + k)(\delta(c_P - 1) + k)} - c_P k\right)}{(\delta - k)^2(c_P - c_D)}$$

The right hand side is strictly greater than one and the condition fails, a contradiction. Therefore, there exists no $\pi \in [\tilde{\pi}_1, \tilde{\pi}_0]$ such that $\xi(x_\phi = P, \delta_i = \delta) \leq \xi(x_\phi = D, \delta_i =$

0). ∎

Proposition A4. In the baseline model of adjudication, random assignment of judges to cases leads to strictly fewer accurate decisions than voluntary assignment. Formally, $\Delta_r < \Delta_v$.

Proof of Proposition A4. This is direct:

$$\begin{aligned}
&\Delta_r < \Delta_v \\
&\quad\iff pq\xi(x^*_\phi,\delta) + p(1-q)\xi(x^*_\phi,0) + (1-p)q\xi(x^*_\phi,0) + (1-p)(1-q)\xi(x^*_\phi,\delta) \\
&\qquad\quad < (1-m)\xi(x^*_\phi,\delta) + m\xi(x^*_\phi,0) \\
&\quad\iff (1-p-q+2pq)\xi(x^*_\phi,\delta) + (q+p-2pq)\xi(x^*_\phi,0) < (1-m)\xi(x^*_\phi,\delta) + m\xi(x^*_\phi,0)
\end{aligned}$$

If $m = q - p$, this reduces to

$$\underbrace{2p(1-q)}_{+}\underbrace{(\xi(x^*_\phi,0) - \xi(x^*_\phi,\delta))}_{-} < 0$$

If $m = p - q$, this reduces to

$$\underbrace{2q(1-p)}_{+}\underbrace{(\xi(x^*_\phi,0) - \xi(x^*_\phi,\delta))}_{-} < 0$$

If $m = p - q$, this reduces to

$$(1-q+p(2q-1))\xi(x^*_\phi,\delta) + (q-p(2q-1))\xi(x^*_\phi,0) < \xi(x^*_\phi,\delta)$$

$$\underbrace{(q(1-p)+p(1-q)}_{+}\underbrace{(\xi(x^*_\phi,0) - \xi(x^*_\phi,\delta))}_{-} < 0$$

Thus, we have shown that $\Delta_r < \Delta_v$. ∎

Lemma A8. With a biased trial judge, there exist two types of equilibria. In these equilibria:

- the appellate court affirms equilibrium judgments in the absence of specific hard information, and corrects errors otherwise;
- the appellant exerts positive effort according to equation (1);
- and the trial judge's equilibrium behavior is as follows:
  - P**-Equilibrium**: for all $\pi \in (\underline{\pi}, \pi_d)$ and $\beta > \hat{\beta}$ or for all $\pi \in (\pi_d, 1)$ and $\beta > 0$, the judge rules in favor of the plaintiff in the absence of information and the judge exerts no effort, where $\underline{\pi} < \frac{1}{2} < \pi_d$, and $\underline{\pi}, \pi_d$ are defined in the proof.
  - D**-Equilibrium**: for all $\pi \in (0, \frac{1}{2})$ and $\beta > 0$ or for all $\pi \in (\frac{1}{2}, \pi_d)$ and $\beta \leq \hat{\beta}$, the judge rules in favor of the defendant in the absence of information and the judge exerts positive effort according to equation (10) in the proof.

Both equilibria are supported by off equilibrium path beliefs and actions described in the proof.

Proof of Lemma A8. First, in all subgames where the uncertainty is resolved for A or L, the equilibrium behavior is as in Proposition A1. Otherwise, there are two kinds of (uninformed) equilibria, one where T rules in favor of P and one where T rules in favor of D.

Candidate equilibrium #1: P-Equilibrium. Suppose A affirms decisions when uninformed (on and off the equilibrium path) and that T does not conceal information in favor of D. Given $x^* = P$, e^* is the solution to the following maximization problem:

$$\max_{e} \quad e\pi\beta + e\varepsilon_T + (1-e)(1 - a_D^*(1-\pi))\beta - ka_D^*(1-\pi) - \frac{c_T}{2}e^2$$

This yields an optimal effort level:

$$e^* = \frac{\beta\pi + \varepsilon_T - \beta(1 - a_D^*(1-\pi))}{c_T} < 0$$

Since ε_T is small, e^* is negative. Moreover since $e \in [0,1]$, we have a corner solution. This allows us to pin down a_D and a_P, which are the same as in Lemma A2.

Next, we check that $x^* = P$ is optimal. Since T exerts no effort, $m_T = \phi$ and thus $x^* = P$ is an equilibrium best response if:

$$U_T(x^* = P) = \beta\Big((1 - a_D) + a_D\pi\Big) - ka_D(1 - \pi) > (\beta - k)a_P\pi = U_T(x' = D)$$

which reduces to

$$\beta > \left(\frac{a_D^*(1-\pi) - a_P^*\pi}{1 - a_D^*(1-\pi) - a_P^*\pi}\right) k \tag{9}$$

Given that $\beta > 0$, equation (9) becomes $\beta > \hat{\beta}$, where

$$\hat{\beta} \equiv \max\left\{0, \left(\frac{a_D^*(1-\pi) - a_P^*\pi}{1 - a_D^*(1-\pi) - a_P^*\pi}\right) k\right\}$$

Because of ε_L and ε_T, it is straightforward to see that the equilibrium messages of L and T are

$$m_T^* = \begin{cases} \omega & \text{if } m_T = \omega \\ \phi & \text{otherwise} \end{cases} \qquad m_L^* = \begin{cases} L & \text{if } s_L = L \\ \phi & \text{otherwise} \end{cases}$$

Next, we verify that A's equilibrium beliefs are consistent. At public history $(x^* = P, m_T^* = m_D^* = \phi)$, $r^* = 0$ is a best response if and only if

$$\frac{\pi(1-e^*)}{\pi(1-e^*) + (1-\pi)(1-a_D^*)} \geq \frac{1}{2}$$

Substituting in $a_D^* = \frac{1-\pi}{c_D}$ and $e^* = 0$

$$0 \geq (c_D - 1)(1 - 2\pi) - \pi^2$$

This reduces to:

$$\pi \geq \underline{\pi} \equiv \sqrt{(c_D - 1)c_D} + 1 - c_D$$

Now, suppose that A reverses off equilibrium path deviations to $x' = D$. Then off the equilibrium path, $a'_P = 0$, and T has no incentive to deviate from the equilibrium for all $\beta > 0$.

Finally, we consider the off equilibrium beliefs of A. We assume that a deviation to $x' = D$ (with no information) leads A to make an inference that T is uninformed since otherwise, T would have revealed her information. (This inference is correct in light of the fact that $e^* = 0$.) If A affirms, its posterior belief off the equilibrium path is thus:

$$\pi^{\text{off}} = \frac{\pi(1 - a'_P)}{\pi(1 - a'_P) + (1 - \pi)}$$

For the belief to be consistent, then:

$$\pi^{\text{off}} = \frac{\pi(1 - a'_P)}{\pi(1 - a'_P) + (1 - \pi)} \leq \frac{1}{2}$$

which reduces to

$$\pi \leq c_P - \sqrt{(c_P - 1)c_P} \equiv \overline{\pi} \in (0.5, 1)$$

Thus, for $\pi \in [\underline{\pi}, \overline{\pi}]$ there exists a P-Equilibrium supported by off equilibrium affirmances and for $\pi \in (\overline{\pi}, 1]$ there exists a P-Equilibrium supported by off equilibrium reversals.

Candidate equilibrium #2: D-Equilibrium. First, suppose that in an equilibrium, A affirms if $x^* = D$ and reverses after a deviation to $x' = P$. As a result, P's optimal effort is given by $a_P^* = \frac{\pi}{c_P}$ as in Proposition A1, and off the equilibrium path $a_D' = 0$.

Next, if T discovers ω, then she is better off setting $x^* = \omega$ and revealing this information. Then, given that T's judgment is $x^* = D$ when uninformed, T's interim utility over effort is:

$$U_T(e) = e\pi(\beta + \varepsilon_T) + e(1-\pi)\varepsilon_T + (1-e)a_P^*\pi(\beta - k) - \frac{c_T}{2}e^2$$

The first order condition yields:

$$e^* = \frac{\pi\beta + \varepsilon_T - a_P^*\pi(\beta - k)}{c_T} = \frac{\pi\beta(c_P - \pi) + \varepsilon_T c_P + \pi^2 k}{c_T c_P} \tag{10}$$

Thus, $e^* > 0$. Because of ε_L and ε_T, it is straightforward to see that the equilibrium messages of L and T are

$$m_T^* = \begin{cases} \omega & \text{if } m_T = \omega \\ \phi & \text{otherwise} \end{cases} \qquad m_P^* = \begin{cases} P & \text{if } s_P = P \\ \phi & \text{otherwise} \end{cases}$$

Finally, we consider A's beliefs. On the equilibrium path, consistency of beliefs requires

$$\Pr(D|h^D) = \frac{(1-\pi)(1-e^*)}{(1-\pi)(1-e^*) + \pi(1-e^*)(1-a_P^*)} \geq \frac{1}{2}$$

which reduces to

$$\pi \leq c_P - \sqrt{(c_P - 1)c_P} \equiv \overline{\pi}$$

Now suppose that A affirms judgments off the equilibrium path: $x' = P$. Then, $a'_D = \frac{1-\pi}{c_D}$ as in Proposition A1 and T has no incentive to deviate if $\beta \leq \hat{\beta}$, as given in equation (9).

Finally, we consider the off equilibrium beliefs of A. By a similar logic as for the P-Equilibrium, we assume that a deviation to $x' = P$ (with no information) leads A to make an inference that T is uninformed. Otherwise, T would have revealed her information. If A reverses, then $a'_D = 0$, and A's posterior belief off the equilibrium path is thus $\pi^{\text{off}} = \pi$. Reversing off the equilibrium path is thus sequentially rational if $\pi \leq \frac{1}{2}$.

However, if $\pi > \frac{1}{2}$, affirming is sequentially rational given $x' = P$ and $a'_D = 0$, but now D has an incentive to exert positive effort, $a'_D > 0$. Then, A's posterior is

$$\pi^{\text{off}} = \frac{\pi}{\pi + (1-\pi)(1-a'_D)}$$

To affirm $x' = P$, the following must hold

$$\pi^{\text{off}} = \frac{\pi}{\pi + (1-\pi)(1-a'_D)} \geq \frac{1}{2}$$

which reduces to

$$\pi \geq \underline{\pi} \equiv \sqrt{(c_D - 1)c_D} + 1 - c_D$$

Since $\underline{\pi} < \frac{1}{2}$, it follows that affirming when $\pi > \frac{1}{2}$ is sequentially rational off the equilibrium path. Finally, for $\pi > 0.5$, we check that $x = D$ is optimal even with off equilibrium affirmances:

$$U_T(x^* = D) = (\beta - k)a_P\pi \geq \beta\Big((1 - a_D) + a_D\pi\Big) - ka_D(1 - \pi) = U_T(x' = P)$$

This is the reverse of the condition for the P-Equilibrium. Therefore, for $\pi \in [0, \frac{1}{2}]$ there exists a D-Equilibrium supported by off equilibrium reversals, and for $\pi \in [\frac{1}{2}, \overline{\pi}]$ and $\beta \leq \hat{\beta}$, there exists a D-Equilibrium supported by off equilibrium affirmances. Note that, for $\pi \in [\tilde{\pi}_d, \overline{\pi}]$, $\hat{\beta} = 0$, so the latter condition can be rewritten: for $\pi \in [\frac{1}{2}, \tilde{\pi}_d]$ and $\beta \leq \hat{\beta}$, there exists a D-Equilibrium supported by off equilibrium affirmances. □

Proposition A5. Suppose that the plaintiff's case is not too strong (*i.e.*, $\pi < \pi'$, where π' is defined in the proof). Then, exists an equilibrium where adjudication by an unbiased judge is lower quality than adjudication by a plaintiff-biased judge. Moreover, if either the judge is not too biased in favor of the plaintiff or the case is a very weak case (or both), then adjudication by an unbiased judge is always lower quality than adjudication by a plaintiff-biased judge.

First, we restate the proposition formally.

Proposition A5′. Let $\pi < \frac{1}{2}$. Then, there exists an equilibrium where $\xi(x_\phi, \beta > 0) > \xi(x_\phi, \beta = 0)$. Moreover, suppose that $\beta < \hat{\beta}$ or that $\pi < \underline{\pi}$ (or both). Then, $\xi(x_\phi, \beta > 0) > \xi(x_\phi, \beta = 0)$ in all equilibria.

Proof of Proposition A5. Existence of a D-Equilibrium is shown in Lemma A8. Moreover, such an equilibrium exists for all $\pi < \pi'$, where

$$\pi' = \begin{cases} \dfrac{1}{2} & \text{if } \beta > \hat{\beta} \\ \pi_d & \text{if } \beta \leq \hat{\beta} \end{cases}$$

Notice that for all $\pi \in [0, \pi']$, the dispassionate judge rules in favor of the defendant. The equilibrium effort of a dispassionate judge is given by equation (5):

$$e_d^* = \frac{\pi^2 k}{c_T c_P}$$

The equilibrium effort of a biased judge is given by equation (10):

$$e_b^* = \frac{\pi^2 k}{c_T c_P} + \frac{\pi\beta(c_P - \pi) + \varepsilon_T c_P}{c_T c_P} > e_d^*$$

In a D-Equilibrium, the quality of decision-making is given by:

$$\xi(x_\phi^*|t) = \begin{cases} e_d^*(D) + [1 - e_d^*(D)][1 - \pi(1 - a_P^*)] & \text{if judge is dispassionate} \\ e_b^*(D) + [1 - e_b^*(D)][1 - \pi(1 - a_P^*)] & \text{if judge is biased} \end{cases}$$

Since $e_b^* > e_d^*$ the latter probability is larger than the former and a biased judge produces higher quality outcomes than a dispassionate judge.

Finally, from Lemma A8, it is apparent that if $\beta < \hat{\beta}$ or $\pi < \underline{\pi}$, then the only equilibria are D-Equilibria. □

Trial Judge's Interior Effort

For T to exert interior effort in equilibrium, the following must hold:

$$c_T > \begin{cases} \frac{1}{c_P}\left[\pi(c_P - \pi)\delta + \pi^2 k\right] & \text{if } x_\phi = D \\ \frac{1}{c_D}\left[(1-\pi)(c_D - (1-\pi))\delta + (1-\pi)^2 k\right] & \text{if } x_\phi = P \end{cases} \tag{11}$$

We define a function $\tilde{c}_T(\pi)$, which returns a threshold value of c_T such that equation (11) holds with equality:

$$\tilde{c}_T(\pi) = \begin{cases} \dfrac{\pi(c_P - \pi)\delta + \pi^2 k}{c_P} & \text{if } \pi \leq \tilde{\pi}_\delta \\ \dfrac{(1-\pi)(c_D - (1-\pi))\delta + (1-\pi)^2 k}{c_D} & \text{if } \pi \geq \tilde{\pi}_\delta \end{cases}$$

There are three properties of this function worth noticing. First, the inequalities in the conditionals are weak. This is because the two components of the piecewise function are equal when evaluated at $\tilde{\pi}_\delta$. Second, $\tilde{c}_T(\pi)$ is strictly increasing when $\pi < \tilde{\pi}_\delta$ and strictly decreasing when $\pi > \tilde{\pi}_\delta$. Thus, $\tilde{c}_T(\pi)$ achieves its global maximum at $\tilde{\pi}_\delta$. Using these facts, in order for T's effort to be interior in in any equilibrium, $c_T > \bar{c}_T$, where:

$$\bar{c}_T \equiv \tilde{c}_T(\pi = \tilde{\pi}_\delta) = \frac{c_P \delta \tilde{\pi}_\delta - (\delta - k)\tilde{\pi}_\delta^2}{c_P} \tag{12}$$

and $\tilde{\pi}_\delta$ is given by equation (4). For the case of a biased judge, replace δ with β in Equation (12).